STAT 18/4/2024

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Hypothesis Testing in Statistical Theory

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Error

✓ Right value → Accept ✗

Right value → Reject ✓

Wrong value → Accept ✓

Wrong value → Reject ✗

Which one is more damaging?

	A	R
✓	N/E	I
✗	II	N/E

A — Accepting wrong (A) ✓

B — Rejecting Correct (B)

Test

degree of freedom

$n-1$

$n-k-1$

t

F

Z χ^2

$a + 2b = 24$
$a + b = 16$
$a - b = 0$
$3a + b = 32$

(2) eqn needed
(4) eqn

df = 4 − 2 = 2

So, any system is Sensible / statistically interpretable

Iff → $\boxed{df \geq 0}$

10%, 5%, 1% almost done

$\boxed{\text{LRT}}$

WHR

0.5–0.9

~~0.4~~ 0.5

0.9

<

~~LHR~~

LRT

Critical Region C (Rejection Region)

$C = \{(x_1, x_2 \ldots x_n) \mid W(x_1, x_2 \ldots x_n) \leq k\}$

$H_1 : \theta = \theta_1$

$H_1 : \theta = \theta_1$

$H_0 : \theta = \theta_0$

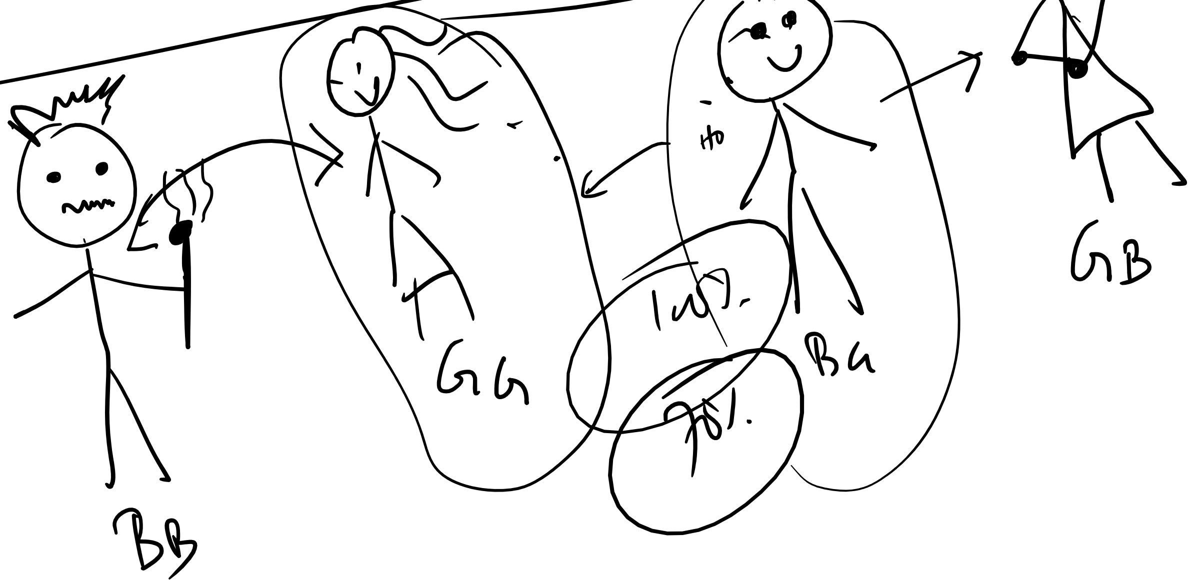
$$CR = \left\{ W \left(x_1, x_2 \ldots x_n \right) = \frac{L(\theta_0, x_1, x_2 \ldots x_n)}{L(\theta_1, x_1, x_2 \ldots x_n)} \le k \right\}$$

	Ho true	Ho fals
Accp Ho	×	II
Reject Ho	I	×

Power of a Test

= 1 − Type II

= 1 −



$P[\text{Rej } H_0 \mid H_0 \text{ true}]$

I

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$P(\text{Acct } H_1 \mid H_0 \text{ is true})$

II

$\alpha \to I$ $\quad 1-\beta =$ Power of the test

$\beta \to II$

Q ISI 2019

$X \to \text{Exp}(\lambda)$ distn

$H_0: \mu = 20$ $\quad \boxed{\mu = \frac{1}{\lambda}}$

$H_1: \mu = 30$

If the value is < 28, H_0 accepted otherwise rejected

$P(\text{Type I})$ & $P(\text{Type II})$??

$\alpha = P(\text{Rej } H_0 \mid H_0 \text{ true})$

$= P(X > 28 \mid X \sim \exp(1/20))$

$= 1 - F_X(28)$

$= e^{-28/20}$

$\beta =$ type II Error $= P[\text{Accept } H_0 \mid H_0 \text{ false}]$

$= P[X < 28 \mid X \sim \exp(1/30)]$

$\Rightarrow F_X(28)$

$= 1 - e^{-28/30}$

$=$ ~~…~~

Type II

Size of the Test & Power of The Test

$f(x) = 3\lambda^2 (\lambda + x)^{-4}$, $\boxed{x > 0}$

In order to test Ho, $\boxed{\lambda = 50}$, H_1 : $\boxed{\lambda = 60}$

Observed value is $\boxed{> 93.5}$ $\rightarrow$ Ho is Rejected

find Size + Power of the Test...

Size $\rightarrow \alpha \rightarrow$ Error (I)

$\alpha = P(\text{Reject Ho} \mid \text{Ho true})$

$= \cancel{P(\text{Reject Ho}}$

$= P(x > 93.5 \mid \lambda = 50)$

$= \int_{93.5}^{\infty} (3 \cdot 50^3 (50 + x)^{-4}\, dx$

$= \left[-50^3 (50 + x)^{-3} \right]_{93.5}^{\infty}$

$\Rightarrow =$

$1 - \int_0^{93.5}$

Power of the Test

$$1-\beta = P(X > 93.5 \mid \lambda = 60)$$

$x < 93.5$

$1 - (x \geq 93.5)$

$$= \int_{93.5}^{\infty} 3 \cdot 60^3 (60+x)^{-4} \, dx$$

$$= \left[-60^3 \left((60+x)^{-3} \right) \right]_{93.5}^{\infty}$$

$X_1, X_2, \ldots, X_{20}$

PDF $f(x;\theta) = \theta^x (1-\theta)^{1-x}$ $\quad x = 0, 1$

$= 0, \quad$ o/w

Where $0 < \theta \leq 1/2$ is a parameter. $\quad H_0: \theta = \frac{1}{2}$ $\quad H_1: \theta < \frac{1}{2}$

If $H_0: \theta < \frac{1}{2}$ is rejected when $\sum_{i=1}^{20} X_i \leq 6$

the P(Type I Error) = ?

Ans: $\sum X_i \sim Bin(20, \theta)$

$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$

$= P\left[\sum x_i \leq 6 \mid H_0 \text{ is true}\right)$

$\mid \theta = \frac{1}{2}$

$$= P\left[\sum x_i \le 6 \,\middle|\, p = \frac{1}{2}\right]$$

$$= \sum_{k=0}^{6} \left({}^{20}C_k\right) \left(\frac{1}{2}\right)^k \left(1-\frac{1}{2}\right)^{20-k}$$

$$= {}^{20}C_1 \left(\frac{1}{2}\right)^1 \left(1-\frac{1}{2}\right)^{19} + {}^{20}C_2 - - - -$$

$\boxed{\text{UNIFORM Distribution Problem}}$

$X \to U(\theta, \theta+1) \quad \theta \in R$

$H_0 : \theta = 1$, $H_1 : \theta = 2$ & has

Critical Region $\{x : x > 1\}$

Power $\to$ ~~1/0/$\frac{1}{2}$/2~~

Size $\to$ ~~4/0/½/2~~

$x \sim U(\theta, \theta+1)$

for $\theta = 1$, $x \sim U(1,2)$

Ans: $\alpha = P(X > 1 \mid \theta = 1)$

$= \int_1^2 1\, dx = 1$

$X \sim U(\theta, \theta+1)$

$\beta = P(x > 1 \mid \theta = 2)$

$= \int_2^3 1\, dx = 1$

$\theta = 2$

$x \sim \underline{U(2,3)}$

~~3 Things~~

1. Identify the Errors
2. Formations of the System

... Distribution

(ii) formations of ...

(iii) Get the Count Distribution

Next Series

Advanced Hypothesis Testing..

Differential Calculus Series

Reason ..

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