Q.1 Firms A and B compete against each other. The products of the firms are differentiated and each month, the firms change their price, the demand functions, facing each firm are,

$$Q_A = 150 - 10P_A + 9P_B$$

$$Q_B = 150 - 10P_B + 9P_A$$

Each firm has a constant MC of \$7/unit.

(a) Find the eq. of reaction function of each firm.

(b) " " Bertrand equilibream price of each firm.

(c) Show reaction functions graphically.

(a) Firm A: $\quad Q_A = 150 - 10P_A + 9P_B \qquad MC_A = 7. \Rightarrow C_A = 7Q_A.$

$$\pi_A = P_A Q_A - 7Q_A = (P_A - 7)\,Q_A$$

$$= (P_A - 7)(150 - 10P_A + 9P_B)$$

$$\frac{\partial \pi_A}{\partial P_A} = 0 \Rightarrow (150 - 10P_A + 9P_B)\cdot 1 + (P_A - 7)(-10) = 0.$$

$$\Rightarrow 150 - 10P_A + 9P_B - 10P_A + 70 = 0.$$

$$\Rightarrow 220 - 20P_A + 9P_B = 0.$$

$$\Rightarrow \boxed{20P_A + 9P_B = 220} \dashrightarrow R_A.$$

Firm B: $\quad Q_B = 150 - 10P_B + 9P_A, \qquad MC_B = 7.$

$$\pi_B = P_B \cdot Q_B - 7Q_B = (P_B - 7)\,Q_B$$

$$= (P_B - 7)(150 - 10P_B + 9P_A)$$

$$\frac{\partial \pi_B}{\partial P_B} = 0 \Rightarrow (150 - 10P_B + 9P_A)\cdot 1 + (P_B - 7)(-10) = 0.$$

$$150 - 10P_B + 9P_A - 10P_B + 70 = 0$$

$$150 - 10P_B + 9P_A - 10P_B + 70 = 0$$

$$220 - 20P_B + 9P_A = 0.$$

$$\boxed{20P_B - 9P_A = 220} \dashrightarrow R_B.$$

(b) $R_A: \quad 20P_A - 9P_B = 220.$

$R_B: \quad 20P_B - 9P_A = 220.$

$$20P_A - 9P_B = 20P_B - 9P_A.$$

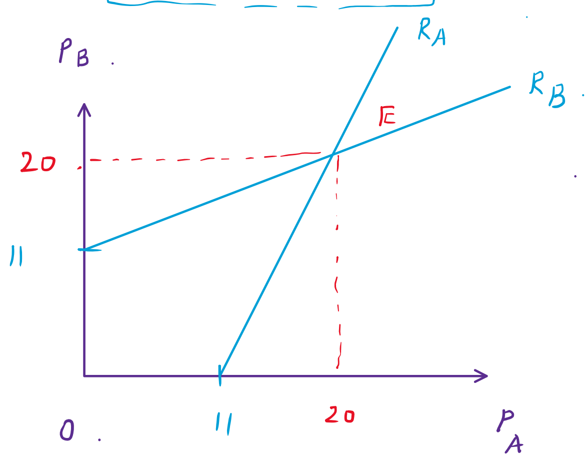
$$29P_A = 29P_B \Rightarrow P_A = P_B.$$

Put in eqn (R_A): $\quad 20P_A - 9P_A = 220.$

$$11P_A = 220 \Rightarrow \boxed{P_A^* = 20 = P_B^*}$$

(c) $R_A: \quad 20P_A - 9P_B = 220.$

$R_B: \quad 20P_B - 9P_A = 220$



Q.2 The market demand curve in an industry of 2 firms is given by Q = 600 − 3P. Each firm has constant MC of \$80/unit.

(a) Find Cournot Equilibrium quantities of each firm

(b) Assuming that firm 2 is a Stackleberg Leader, find out equilibrium quantities for each firm.

(c) Calculate & compose, the profit of each firm, under Cournot & Stackleberg Equilibria. Under which equilibrium, the industry profit is ~~highlight~~ highest & why?

(a) $q_1^* = 120 \quad q_2^* = 120 \quad p^* = 120 \quad \pi_1^* = 13440 = \pi_2^*$

(b) $Q = 600 - 3P$, $MC_1 = MC_2 = 80$, Firm II is the Leader.

$$3P = 600 - Q \Rightarrow P = 200 - \frac{1}{3}(q_1 + q_2)$$

Firm I is the follower:

$$\pi_1 = Pq_1 - C_1 = \left[200 - \frac{1}{3}(q_1+q_2)\right]q_1 - 80q_1$$

$$\pi_1 = 200q_1 - \frac{1}{3}(q_1^2 + q_1q_2) - 80q_1$$

$$\pi_1 = 120q_1 - \frac{1}{3}(q_1^2 + q_1q_2)$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow 120 - \frac{1}{3}(2q_1 + q_2) = 0$$

$$\Rightarrow 120 = \frac{1}{3}(2q_1 + q_2)$$

$$\Rightarrow 360 = 2q_1 + q_2 \Rightarrow q_1 = \frac{1}{2}(360 - q_2) = 180 - \frac{q_2}{2}$$

$$\Rightarrow \boxed{q_1 = 180 - \frac{q_2}{2}} \rightarrow R_1$$

Firm II is the leader:

$$\pi_2 = Pq_2 - C_2 = \left[200 - \frac{1}{3}(q_1+q_2)\right]q_2 - 80q_2$$

$$\pi_2 = 200q_2 - \frac{1}{3}(q_1q_2 + q_2^2) - 80q_2$$

$$\pi_2 = 120q_2 - \frac{1}{3}(q_1q_2 + q_2^2)$$

Replacing R_1 in π_2:

$$\pi_2 = 120q_2 - \frac{1}{3}\left[\left(180 - \frac{q_2}{2}\right)q_2 + q_2^2\right]$$

$$\pi_2 = 120q_2 - \frac{1}{3}\left[180q_2 - \frac{q_2^2}{2} + q_2^2\right]$$

$$\pi_2 = 120\,q_2 - \frac{1}{3}\left[180\,q_2 - \frac{q_2^2}{2} + q_2^2\right]$$

$$\pi_2 = 120\,q_2 - 60\,q_2 - \frac{1}{3}\left(\frac{q_2^2}{2}\right)$$

$$\pi_2 = 60\,q_2 - \frac{1}{6}\cdot q_2^2 .$$

$$\frac{d\pi_2}{dq_2} = 0 \Rightarrow 60 - \frac{1}{3}\cdot q_2 = 0 \Rightarrow q_2^* = 180 .$$

$$\therefore\ q_1^* = 180 - \frac{q_2}{2} = 180 - 90 = 90 .$$

$$p^* = 200 - \frac{1}{3}(q_1^* + q_2^*) = 110 .$$

$$\pi_1^* = 110 \times 90 - 80 \times 90 = 30 \times 90 = 2700 .$$

$$\pi_2^* = 110 \times 180 - 80 \times 180 = 180 \times 30 = 5400 .$$

		(q)	$(1-q)$
	I \ II	c	d
(p)	a	$\boxed{(1,1)}$	$(0,2)$
$(1-p)$	b	$(2,0)$	$\boxed{(1,1)}$

Pure-strategy NE:

I → a , II → c

(a, c) is the NE.

$\boxed{(a, c)}$ (b, d)

Mixed strategy NE : $\{p^*, q^*\}$.