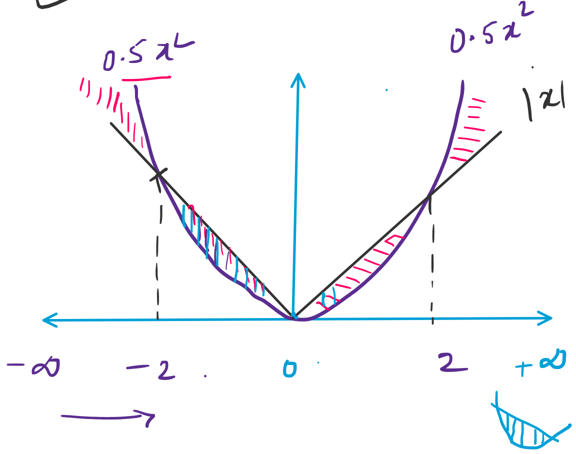
Q. The no. of disjoint intervals over which $f(x) = \left|0.5x^2 - |x|\right|$ is decreasing is: (a) 1 (b) 2 ~~(c)~~ 3 (d) None.

$f(x) = \left| 0.5x^2 - |x| \right|$

$\boxed{0.5x^2 - |x|}$



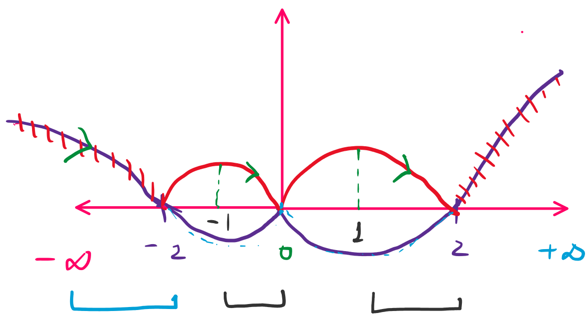
$0.5x^2 = x$.

$x = \frac{1}{0.5} = 2$

$0.5x^2 = -x$.

$x = -\frac{1}{0.5} = -2$

No. of intervals for decreasing $f(x) = 3$.



Mean Value Theorems :-

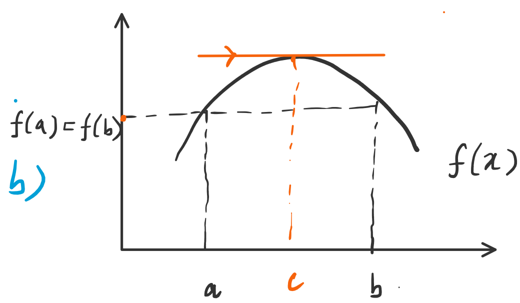
Rolle's Theorem :-

Consider a fn $f(x)$ over $[a, b]$.

(i). If $f(x)$ is continuous over $[a, b]$.

(ii) If $f(x)$ is differentiable over (a, b)

(iii) $f(a) = f(b)$



Then $\exists$ a pt 'c' $\in (a, b)$ s.t $f'(c) = 0$.

Lagrange's Theorem :-

Consider

Lagrange's Theorem :-

Consider a fn $f(x)$ over $[a, b]$.

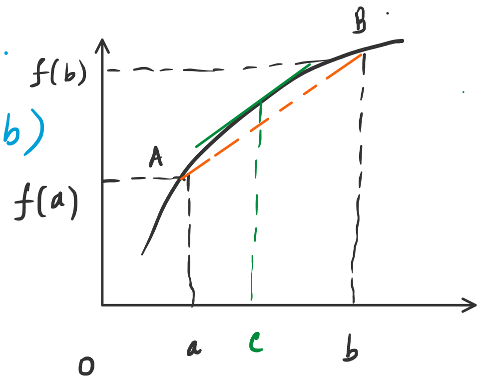
(i) If $f(x)$ is continuous in $[a, b]$.

(ii) If $f(x)$ is differentiable in (a, b)

Then $\exists$ a pt 'c' $\in (a, b)$ s.t

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

↳ slope of the line joining pts A, B



Interpretation:

Rolle's Th: $\exists$ atleast one pt 'c' $\in (a, b)$ where tangent is parallel to the x-axis.

Lagrange's Th: $\exists$ atleast one pt 'c' $\in (a, b)$ s.t slope of the line segment AB equals slope of the tangent at that point.

Q. If $f(x)$ and $g(x)$ are differentiable fns in $[0, 1]$ s.t $f(0) = 2$, $f(1) = 6$, $g(0) = 0$, $g(1) = 2$. Then show that $\exists$ a pt 'c' $\in (0, 1)$ s.t $f'(c) = 2g'(c)$.

Note: To prove: $f'(c) = 2g'(c)$

$$\Rightarrow \boxed{f'(c) - 2g'(c)} = 0.$$

↳ $h'(c) = 0$ [Proving Rolle's Thm

$h(x) = f(x) - 2g(x)]$.

Let $h(x) = f(x) - 2g(x)$. $[0,1]$.

(i) $f(x), g(x)$ are continuous in $[0,1] \Rightarrow h(x)$ is cont in $[0,1]$

(ii) $f(x), g(x)$ are diff in $(0,1) \Rightarrow h(x)$ is diff in $(0,1)$

(iii) $h(0) = f(0) - 2g(0) = 2$

$h(1) = f(1) - 2g(1) = 2$

$\therefore$ Rolle's Theorem is applicable, i.e $\exists$ a pt 'c' $\in (0,1)$

s.t $h'(c) = 0$.

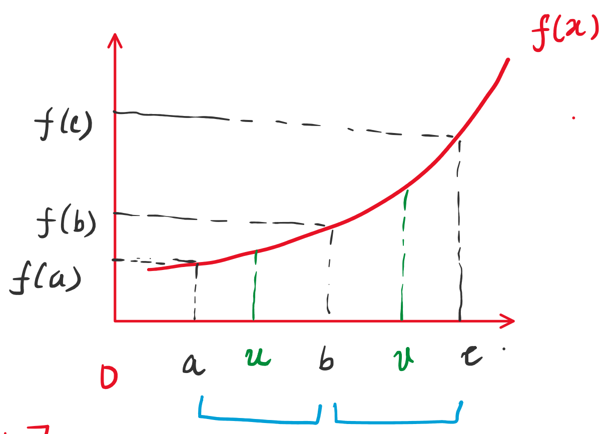
$$f'(c) - 2g'(c) = 0 \Rightarrow \boxed{f'(c) = 2g'(c),\ 0 < c < 1}$$

Q. Let $a, b, c \in \mathbb{R}$ s.t $a < b < c$. A fn $f(x)$ is continuous in $[a,c]$ and diff in (a,c) and $f'(x)$ is strictly increasing in (a,c). Prove that $(c-b)f(a) + (b-a)f(c) > (c-a)f(b)$

$f'(x)$ strictly increasing

$\Rightarrow \frac{d}{dx} f'(x) > 0 \Rightarrow f''(x) > 0$.

$f(x)$ is convex.



Apply Lagrange's Th on $f(x)$ in $[a,b]$:

$\therefore \exists$ a pt $u \in (a,b)$ s.t $f'(u) = \frac{f(b) - f(a)}{b - a}$.

Apply Lagrange's Th on $f(x)$ in $[b,c]$:

Apply Lagrange's Th on $f(x)$ in $[b,c]$:

$\therefore \exists$ a pt $v \in (b,c)$ s.t $f'(v) = \frac{f(c)-f(b)}{c-b}$.

$\therefore \quad f'(u) < f'(v) \quad [\because f'(x)$ is strictly increasing$]$

$$\frac{f(b)-f(a)}{b-a} < \frac{f(c)-f(b)}{c-b}.$$

$$[f(b)-f(a)](c-b) < [b-a].f(c)-f(b).$$

$$f(b)(c-b) - (c-b)f(a) < (b-a)f(c) - (b-a)f(b)$$

$$f(b)[c-\cancel{b}+\cancel{b}-a] - (c-b)f(a) < (b-a)f(c)$$

$$(c-a)f(b) - (c-b)f(a) < (b-a)f(c).$$

$$(c-a)f(b) < (b-a)f(c) + (c-b)f(a)$$

HW.

Q. If $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$ has a positive root 'α', then, $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \cdots + a_1 = 0$ has a positive root which is:

(a) $> \alpha$ (b) $< \alpha$ (c) $\geq \alpha$ (d) $= \alpha$.