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27 February 2024 17:24

Bounds of a Sequence : Bounded Sequence

limits

lower upper

$\{1, 2, 3, 4, 5, 6\}$ Bounded.

Lower bounded $\{x, x \in N\} = \{1, 2, 3, \ldots\}$

Unbounded $\{x, x \in Z\} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

Upper bounded. $\{x, x \in Z, x \leq 10\}$

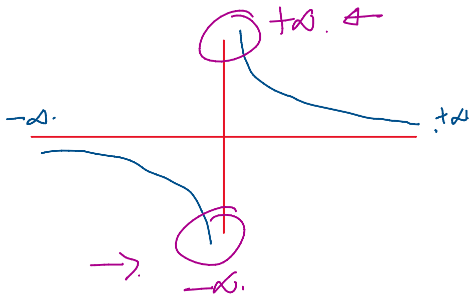
oscillatory. $\{x : |x| = 1\} = \{-1, 1, -1, 1 \ldots\}$

$y = \frac{1}{x}$. $x \in R$

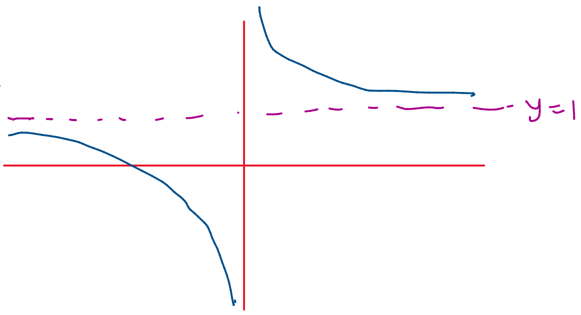
$x \to +\infty$ $y \to 0$

$x \to -\infty$ $y \to 0$

x	y
-1000	-0.001
-10000	-0.0001
-100000	-0.00001
$\vdots$	$\vdots$

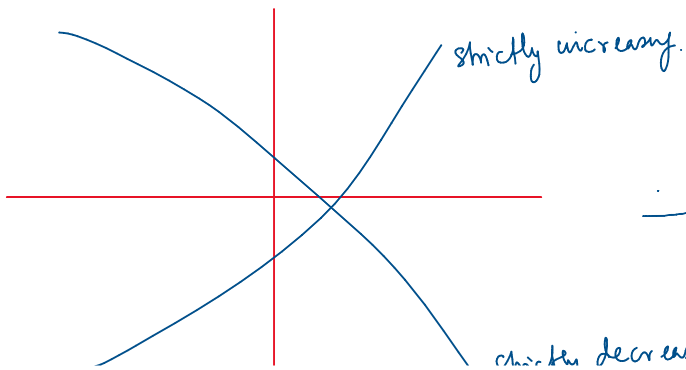


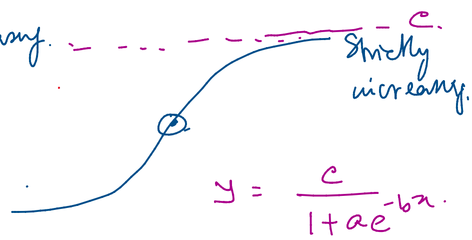
$y = 1 + \frac{1}{x}$.



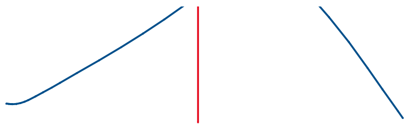
1.8. Monotonic Sequence.

$f'(x) > 0$ OR $f'(x) < 0$

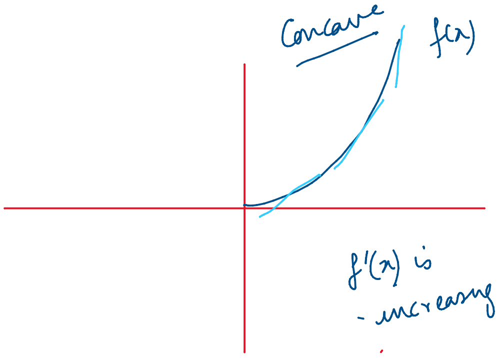


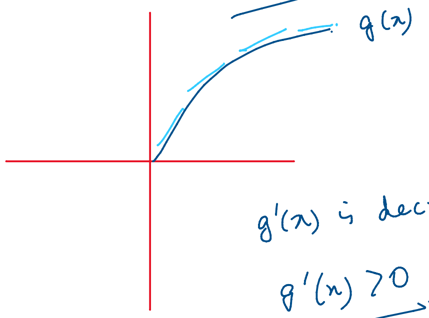


$y = \frac{c}{1 + ae^{-bx}}$.



Strictly decreasing





$f'(x)$ is increasing.

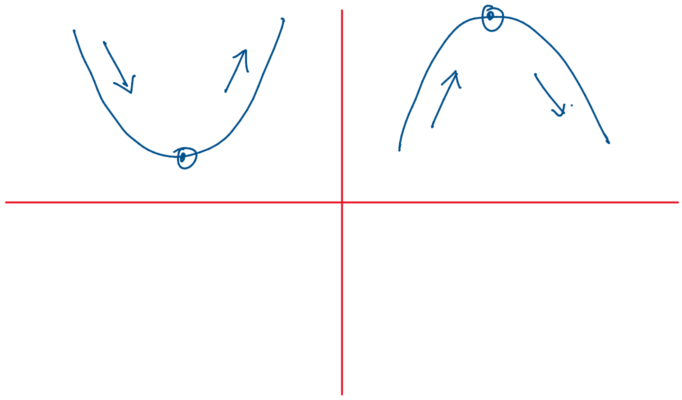
$f'(x) > 0$

$f''(x) > 0$

$g'(x)$ is decreasing

$g'(x) > 0$

$f''(x) < 0$.



Monotonic sequences. → (1) $f'(x) > 0$ or $f'(x) < 0$

$\Downarrow$

$f'(x) \neq 0$

$\Downarrow$

No maxima or minima.

Monotonic sequences ↓ One-one and onto. (Bijective)

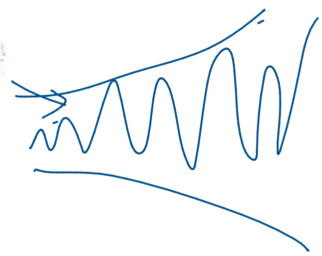
Convergent Sequence.

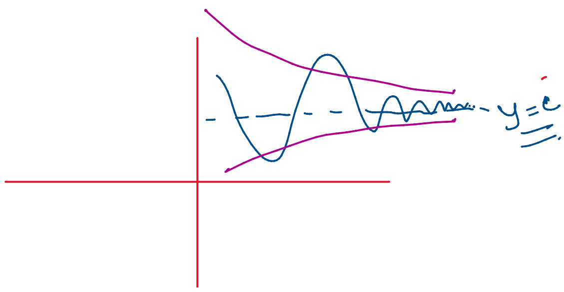
concept of limit

↓

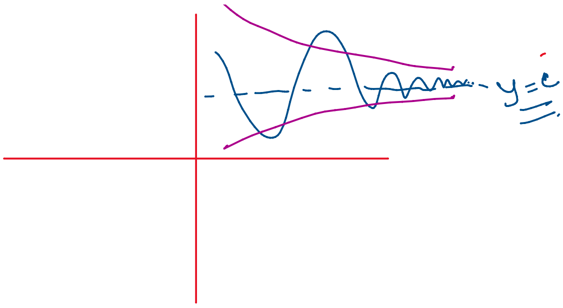
sequence moves towards a finite value as $x \to \infty$

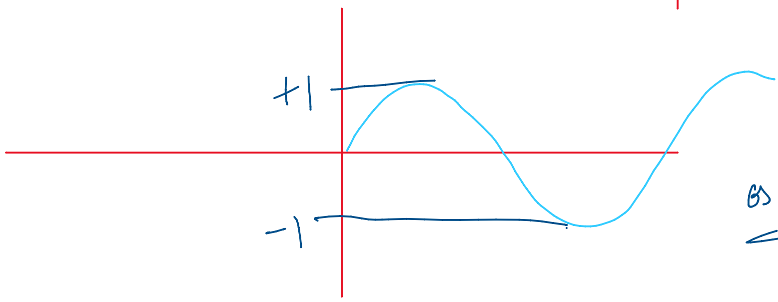
Non-Covergent Sequence.





sequence moves towards a finite value as $x \to \infty$





non convergent

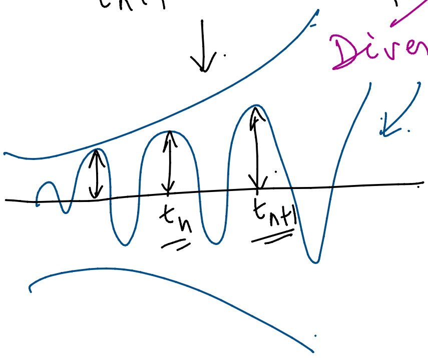
oscillatory

Non convergence

$t_{n+1} > t_n$

Divergence.

Oscillatory

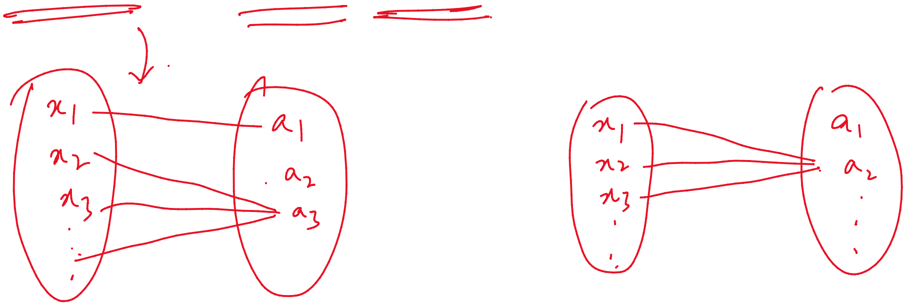


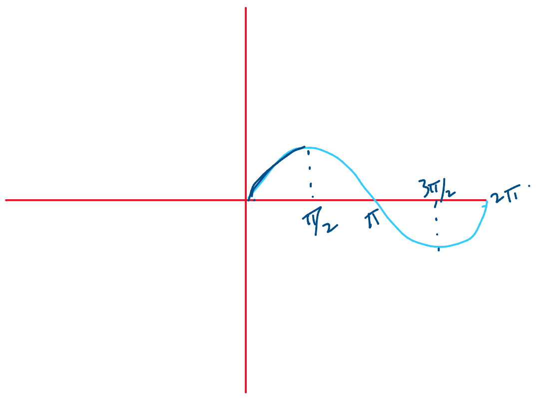
$0, 1, 0, 1, 0, 1 \ldots \ldots$

$t_{n+1} < t_n$

$\frac{t_{n+1}}{t_n} < 1$

Pointwise and uniform convergence of series of functions





$[0, \pi/2] \to$ monotonic.

$[0, \pi] \to$ non-monotonic.

domain.

$x \to$ +ve fraction

Suppose that $f_n : \boxed{(0,1)} \to \mathbb{R}$ is defined by

$$f_n(x) = \frac{n}{nx+1}. \quad = \frac{1}{x + \frac{1}{n}}.$$

$$\lim_{n \to \infty} f_n(x) = \frac{1}{x}.$$

$$f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1, \\ 1 & \text{if } x = 1. \end{cases}$$

$[0, 1]$.

$$\lim_{n\to\infty} f_n(x) = \begin{cases} 0 & x = [0,1) \\ 1. & x = 1 \end{cases}$$