Testing of Hypothesis

Case 1: μ unknown and σ known

$H_0 : \mu = \mu_0$

$$Z \text{ or } \tau = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

(i) $H_1 : \mu \neq \mu_0$ (Two-tail test)

H_0 is rejected if $|\tau| > \tau_{\alpha/2}$.

(ii) $H_1 : \mu > \mu_0$ (one-tail → right side)

H_0 is rejected if $|\tau| > \tau_\alpha$

(iii) $H_1 : \mu < \mu_0$ (one-tail → left side)

H_0 is rejected if $|\tau| < \tau_{1-\alpha}$ $(-\tau_\alpha)$

$|\tau|$ $\alpha/2$ χ^2 $\ominus$ $\oplus$

Case 2: μ known and σ unknown.

$H_0 : \sigma = \sigma_0$

$$\sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma_0} \right)^2 = \frac{\Sigma (x_i - \mu)^2}{\sigma_0^2} \sim \chi^2_{(n)}$$

(i) $H_1 : \sigma \neq \sigma_0$

H_0 is rejected if $\chi^2 < \chi^2_{1-\frac{\alpha}{2}, n}$

or $\chi^2 > \chi^2_{\alpha/2, n}$

(ii) $H_1 : \sigma > \sigma_0$ (right tail test)

H_0 is rejected if $\chi^2 > \chi^2_{\alpha, n}$

(iii) $H_1 : \sigma < \sigma_0$ (left tail test)

(iii) $H_1 : \sigma < \sigma_0$ (left tail test)

H_0 is rejected if $\chi^2 < \chi^2_{1-\alpha, n}$

Case III : μ and σ both unknown

$H_0 : \mu = \mu_0$ — s or $s' \Rightarrow$ sample s.d

$$t = \frac{\bar{x} - \mu_0}{s'/\sqrt{n}} \sim t_{(n-1)}.$$

(i) $H_1 : \mu \neq \mu_0 \Rightarrow H_0$ is rejected if $|t| > t_{\frac{\alpha}{2}, (n-1)}$

(ii) $H_1 : \mu > \mu_0 \Rightarrow H_0$ is rejected if $t > t_{\alpha, (n-1)}$

(iii) $H_1 : \mu < \mu_0 \Rightarrow H_0$ is rejected if $t < -t_{\alpha, (n-1)}$.

Case IV : $H_0 : \sigma = \sigma_0$

$$\chi^2 = \frac{\sum (x_i - \bar{x})^2}{\sigma_0^2} \sim \chi^2_{(n-1)}.$$

$$= \frac{(n-1) s'^2}{\sigma_0^2} \sim \chi^2_{(n-1)}$$

(i) $H_1 : \sigma \neq \sigma_0 \Rightarrow H_0$ is rejected if $\chi^2 > \chi^2_{\frac{\alpha}{2}, (n-1)}$

or, $\chi^2 < \chi^2_{1-\frac{\alpha}{2}, (n-1)}$

(ii) $H_1 : \sigma > \sigma_0 \Rightarrow \ldots\ldots \chi^2 > \chi^2_{\alpha, (n-1)}$

(iii) $H_1 : \sigma < \sigma_0 \Rightarrow \ldots\ldots \chi^2 < \chi^2_{1-\alpha, (n-1)}$.

Comparison of two normal distributions

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sample 1 $\Rightarrow \bar{x}_1, s_1, s_1' \Leftarrow \mu_1, \sigma_1$

2 $\Rightarrow \bar{x}_2, s_2, s_2' \Leftarrow \mu_2, \sigma_2$

Case 1: μ_1 and μ_2 unknown with (σ_1 and σ_2 known).

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad z\frac{(\bar{x}_1 - \bar{x}_2) - (\lambda)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$H_0: \mu_1 = \mu_2$ or $\mu_1 - \mu_2 = \lambda$

(i) $H_1: \mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq \lambda$

$\Rightarrow$ H_0 is rejected $\Rightarrow |z| > z_{\alpha/2}$

(ii) $H_1: \mu_1 - \mu_2 > \lambda$

same as above in case (i)

(iii) $H_1: \mu_1 - \mu_2 < \lambda$

Case 2: μ unknown, σ unknown

$H_0: \mu_1 - \mu_2 = \lambda$

$$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1'^2}{n_1} + \frac{s_2'^2}{n_2}}} \sim t_{(n_1 + n_2 - 2)}$$

$H_1: \mu_1 - \mu_2 \neq \lambda \Rightarrow H_0$ rejected if $|t| > t_{\frac{\alpha}{2}, (n_1+n_2-2)}$

$H_1: \mu_1 - \mu_2 > \lambda \Rightarrow$ . . . . case 2 above.

$H_1: \mu_1 - \mu_2 < \lambda \Rightarrow$ . . . . . .

δ $\qquad \frac{\Sigma(x_i - \mu_1)}{\sigma_p^2}$

Case 3: σ_1 unknown, μ_1 and μ_2 known.

and σ_2 $\qquad H_0: \sigma_1 = \sigma_2$ i.e. $\frac{\sigma_1}{\sigma_2} = 1$

or $\frac{\sigma_1}{\sigma_2} = \lambda$

Test statistics.

$$\frac{\sum_{i=1}^{n} (x_i - \mu_1)^2 / n_1}{\sum_{i=1}^{n} (x_i - \mu_2)^2 / n_2} \times \frac{\sigma_2^2}{\sigma_1^2}$$

$$\frac{\Sigma (x_i - \mu_1)^2 / n_1}{\Sigma (x_i - \mu_2)^2 / n_2} \times \frac{1}{\lambda^2} \sim F_{n_1, n_2}$$

(i) $H_1: \frac{\sigma_1}{\sigma_2} \neq \lambda \Rightarrow H_0$ is rejected if $F > F_{\frac{\alpha}{2}; (n_1, n_2)}$

or $F < F_{1-\frac{\alpha}{2}; (n_1, n_2)}$

(ii) $H_1: \frac{\sigma_1}{\sigma_2} > \lambda \Rightarrow$ . . . . if $F > F_{\alpha; (n_1, n_2)}$

(iii) $H_1: \frac{\sigma_1}{\sigma_2} < \lambda \Rightarrow$ . . . . if $F < F_{1-\alpha; (n_1, n_2)}$.

Case IV σ_1, σ_2 unknown as well as μ_1 and μ_2 unknown.

Case IV σ_1, σ_2 unknown. unknown.

$H_0: \frac{\sigma_1}{\sigma_2} = \lambda$

$$F = \frac{\sum_{i=1}^{m} (x_i - \bar{x}_1)^2 / (n_1 - 1)}{\sum_{i=1}^{n} (x_i - \bar{x}_2)^2 / (n_2 - 1)} \times \frac{\sigma_2^2}{\sigma_1^2}$$

$$= \frac{s_1'^2}{s_2'^2} \times \frac{1}{\lambda^2} \sim F_{n_1 - 1, n_2 - 1}.$$

Q1: Suppose that the time taken by a particle to move from one fixed point to another fixed point is distributed as $N(\mu, \sigma^2 = 4)$.
A random sample of 9 readings has a mean of 50. Test the hypothesis $H_0: \mu = 52$ against $H_1: \mu \neq 52$ at 1% level.

Soln: Given: μ = unknown
$\sigma^2 = 4$, $\sigma = 2$
$n = 9$
$\alpha = 1\% = 0.01$
$\bar{x} = 50$

We have to test
$H_0: \mu = 52$
vs $H_1: \mu \neq 52$

$\therefore$ The test statistics under H_0 is $\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \frac{50 - 52}{2/\sqrt{9}}$

$= \frac{-\cancel{2}}{\cancel{2}} \times 3$

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$\therefore z_{0.005} = 2.58$

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$\overline{\; z \;}$

$= -3$

Since $3 > 2.58$

i.e. $|z| > z_{0.005}$ $\Rightarrow$ H_0 is rejected

$\Rightarrow \mu \neq 52$

at 1% level of significance

Q2 A company has been producing steel tubes of mean diameter 2.00 cm.
A sample of 10 tubes gives a mean diameter of 2.01 cm and variance of 0.004 sq cm.
Is the difference in the mean value significant?

$H_0: \mu = 2.00$ cm

v/s $H_1: \mu \neq 2.00$ cm

Given; $\bar{x} = 2.01$ cm

$s^2 = 0.004$

$s = 0.0632$

$n = 10$

$s'^2 = \frac{n}{n-1} s^2$

$= \frac{10}{9} \times 0.004$

$s' = \frac{\sqrt{10}}{3} \times 0.2$

$t = \frac{\bar{x} - \mu}{s'/\sqrt{n}}$

$\alpha = 5\% = 0.05$

$t = \frac{(2.01 - 2)\cancel{\sqrt{10}}}{\frac{\cancel{\sqrt{10}}}{3} \times 0.0632}$

$t_{\frac{\alpha}{2}, n-1} = t_{0.025, 9}$

$= 2.262$

$= \frac{0.01}{0.0632} \times 3 = \frac{0.03}{0.0632} = 0.474$

Since $0.474 < 2.262$

i.e $t < t_{\frac{\alpha}{2}, n-1}$

then H_0 is accepted at 5% level of significance

So we conclude that the diff in mean value is not significant.

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