Q. Let $f: \mathbb{R} \to \mathbb{Z}$ such that $f(x) = \lceil x \rceil$ (Least Integer fn)

Define: $g(x) = |f(x)| - f(|x|)$. Then R_g:

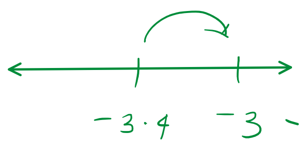
(a) $\{0, 1\}$ (c) $\{-1, 0, 1\}$

(b) $[-1, 1]$ (d) $\{-1, 0\}$.

If $x \in \mathbb{Z}$, $g(x) = 0$.

Eg: $x = 2.3$, $\lceil x \rceil = 3 = f(x)$

$$g(x) = |3| - f(|3|) = 0.$$



$x = -3.4$, $\lceil x \rceil = -3 = f(x)$

$$g(x) = |-3| - f(3.4)$$
$$= 3 - \lceil 3.4 \rceil$$
$$= 3 - 4 = -1.$$

$\therefore$ $R_f \in \{-1, 0\}$

Q. Range of the function $f(x) = 4^x + 4^{-x} + 2^x + 2^{-x} + 3$ is:

(a) $(-\infty, -7]$ (b) $[7, \infty)$ (c) $(-\infty, -6]$ (d) $[6, \infty)$.

$$f(x) = \underbrace{(4^x + 4^{-x})} + \underbrace{(2^x + 2^{-x})} + 3 \geq 2 + 2 + 3 = 7.$$

4^x, 4^{-x}, AM $\geq$ GM.

$$\left(\frac{4^x + 4^{-x}}{2}\right) \geq \sqrt{\cancel{4^x} \cdot \cancel{4^{-x}}} \Rightarrow 4^x + 4^{-x} \geq 2$$

2^x, 2^{-x}, AM $\geq$ GM.

$$\frac{2^x + 2^{-x}}{2} \geq \sqrt{2^x \cdot 2^{-x}} \Rightarrow 2^x + 2^{-x} \geq 2.$$

$\therefore$ $R_f \in [7, \infty)$.

$\therefore \; R_f \in [7, \infty)$.

Q. If $f(x) = \ln(6 - |x^2 + x - 6|)$, then the domain of $f(x)$ contains how many integral values of x?

(a) 5 $\quad$ ~~(b)~~ 4 $\quad$ (c) ∞ $\quad$ (d) None.

$$6 - |x^2 + x - 6| > 0.$$

$$|x^2 + x - 6| < 6.$$

$$-6 < x^2 + x - 6 < 6.$$

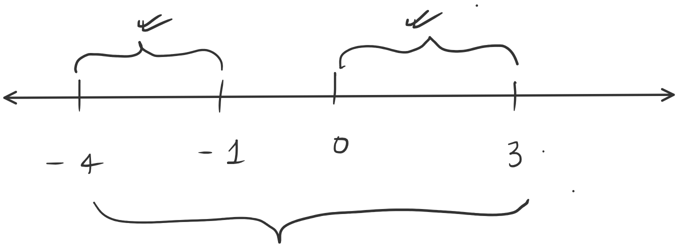
$x^2 + x - \cancel{6} > -\cancel{6}$ $\quad$ or $\quad$ $x^2 + x - 6 < 6$.

$x^2 + x > 0$. $\quad$ $x^2 + x - 12 < 0$.

$x(x+1) > 0$. $\quad$ $(x+4)(x-3) < 0$.

$\hookrightarrow x > 0$ or $x < -1$. $\quad$ $x \in (-4, 3)$.

$\therefore \; D_f : (-4, -1) \cup (0, 3)$.



Integers: $-3, -2, 1, 2$.

$\Rightarrow$ 4 integers.

Q. If $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(x+y) = f(x) + f(y) \; \forall x, y \in \mathbb{R}$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r) = ?$

$$f(2) = f(1) + f(1) = 2f(1).$$

$$f(3) = f(2) + f(1) = 3f(1).$$

$$\cdots\cdots\cdots\cdots$$

$$f(n) = n f(1).$$

$$\therefore \sum_{r=1}^{n} f(r) = f(1) + f(2) + \cdots + f(n)$$

$$= f(1) + 2\cdot f(1) + \cdots + n\cdot f(1)$$

$$= f(1)\,[1 + 2 + \cdots + n]$$

$$= f(1)\cdot\frac{n(n+1)}{2} = \frac{7n(n+1)}{2}$$

Q. Let $f(x) = \sum_{r=0}^{100} \alpha_r x^r$ and $f(0)$ and $f(1)$ are odd numbers; then for any integer x :-

(a) $f(x)$ is odd/even no. if x is odd/even no.

(b) $f(x)$ is odd/even no. if x is even/odd no.

(c) $f(x)$ is even no. $\forall x$.

(d) $f(x)$ is odd no. $\forall x$.

$$\left.\begin{aligned} f(0) &= \alpha_0 \\ f(1) &= \alpha_0 + \alpha_1 + \cdots + \alpha_{100} \end{aligned}\right\} \Rightarrow \text{odd numbers.}$$

Let x be even. $x = 2n$.

$$f(2n) = \alpha_0 + \alpha_1(2n) + \alpha_2(2n)^2 + \cdots + \alpha_{100}(2n)^{100}.$$

$$= \underset{\text{odd}}{\alpha_0} + \underbrace{2[\cdots\cdots]}_{\text{even}} = \text{odd no.}$$

Let x be odd, $x = (2n+1)$

$$f(2n+1) = \alpha_0 + \alpha_1(2n+1) + \alpha_2(2n+1)^2 + \cdots + \alpha_{100}(2n+1)^{100}$$

$$= \alpha_0 + \alpha_1(2n+1) + \alpha_2(1 + 4n + 4n^2) + \cdots +$$

$$\alpha_{100}(1 + \text{even no.})$$

$$= \overbrace{(\alpha_0 + \alpha_1 + \alpha_2 + \cdots + \alpha_{100})} + [2n(\alpha_1) + \alpha_2(4n + 4n^2)$$

$+ \cdots + \alpha_{100}$ (even no.) $]$.

odd + even = odd no.

Q. If $f(x) = [x]^2 - [x^2]$ where $[x]$ is the GIF and $x \in [0,2]$. Find the range of $f(x)$.

$x \in [0, 2]$.

$$[x] = \begin{cases} 0, & x \in [0, 1) \\ 1, & x \in [1, 2) \\ 2, & x = 2 \end{cases}$$

$$[x^2] = \begin{cases} 0, & 0 \le x^2 < 1 \Rightarrow x \in [0,1) \\ 1, & 1 \le x^2 < 2 \Rightarrow x \in [1, \sqrt{2}) \\ 2, & 2 \le x^2 < 3 \Rightarrow x \in [\sqrt{2}, \sqrt{3}) \\ 3, & 3 \le x^2 < 4 \Rightarrow x \in [\sqrt{3}, 2) \\ 4, & x = 2 \end{cases}$$