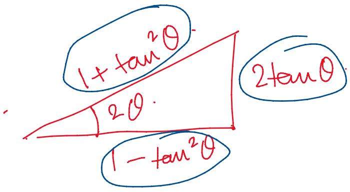
07 May 2024 11:43

Evaluate ∫ 1/sin x (2 + cos x – 2 sinx) dx.

$$I = \int \frac{1}{\sin x(2+\cos x - 2\sin x)}\,dx.$$



$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}.$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} \qquad \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\sin x = \frac{2\tan(x/2)}{1+\tan^2(x/2)} \qquad \cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$$

$$I = \int \frac{1}{\frac{2\tan(x/2)}{1+\tan^2(x/2)}\left[2+\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)} - 2\cdot\frac{2\tan(x/2)}{1+\tan^2 x/2}\right]}\,dx.$$

$$= \int \frac{\sec^2 x/2 \times \sec^2 x/2}{2\tan(x/2)\left[2+2\tan^2 x/2 + 1 - \tan^2 x/2 - 4\tan x/2\right]}\,dx.$$

$3-4t+t^2$
$=(3-3t)-(t-t^2)$
$=3(1-t)-t(1-t)$
$=(3-t)(1-t)$
$=(t-3)(t-1)$

$$= \int \frac{\sec^2 x/2\ \sec^2 x/2\ dx}{2\tan x/2\left[3-4\tan x/2+\tan^2 x/2\right]}$$

$\tan x/2 = t.$

$\frac{1}{2}\sec^2 x/2\,dx = dt.$

$\sec^2 x/2\,dx = 2\,dt.$

$$= \int \frac{(1+t^2)\cdot\cancel{2}\,dt}{\cancel{2}t\left[3-4t+t^2\right]} = \int \boxed{\frac{(1+t^2)\,dt}{t(t-1)(t-3)}}$$

factors in the denominator

⇓

partial fractions

$$\frac{(1+t^2)}{t(t-1)(t-3)} = \frac{A}{t} + \frac{B}{t-1} + \frac{C}{t-3}$$

$$(1+t^2) = A(t-1)(t-3) + Bt(t-3) + Ct(t-1)$$

$t=0$: $1 = A\times 3 \rightarrow A = 1/3.$

$t=1$: $2 = -2B \rightarrow B = -1$

$t=3$: $10 = 6C \rightarrow C = \frac{5}{3}.$

$$I = \int \frac{(1/3)}{t}\,dt + \int \frac{(-1)\,dt}{t-1} + \int \frac{(5/3)\,dt}{t-3}$$

$$= \frac{1}{3}\log|t| - \log|t-1| + \frac{5}{3}\log|t-3| + c.$$

$$= \frac{1}{3}\left[\log|t| - 3\log|t-1| + 5\log|t-3|\right] + c.$$

$$= \frac{1}{3}\left[\log|t| - 3\log|t-1| + 5\log|t-3|\right] + c.$$

$$= \frac{1}{3}\log\left[\frac{t\times|t-3|^5}{|t-1|^3}\right] + c.$$

$$= \frac{1}{3}\log\left[\frac{\tan(x/2)\,|\tan x/2 - 3|^5}{|\tan x/2 - 1|^3}\right] + c.$$

$$\int\left(\frac{2a+x}{a+x}\right)\sqrt{\frac{a-x}{a+x}}\,dx =$$

how to get rid of the $\sqrt{\ }$?

$\tan\theta = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ $\quad \cos 2\theta = \frac{x}{a}$.

$\boxed{x = a\cos 2\theta}$ $\quad dx = -2a\sin 2\theta\, d\theta$.

$\boxed{\begin{array}{l} 1-\cos 2\theta = 2\sin^2\theta \\ 1+\cos 2\theta = 2\cos^2\theta \end{array}}$

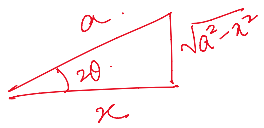
$$I = \int\left(\frac{2a + a\cos 2\theta}{a + a\cos 2\theta}\right)\sqrt{\frac{a - a\cos 2\theta}{a + a\cos 2\theta}}\left(-2a\sin 2\theta\, d\theta\right) = -2a\int\frac{(2+\cos 2\theta)}{1+\cos 2\theta}\cdot\tan\theta\,\sin 2\theta\, d\theta$$

$$= -2a\int\frac{(2+\cos 2\theta)}{\cancel{2}\cos^2\theta}\cdot\tan\theta\cdot\cancel{2}\sin\theta\cos\theta\, d\theta = -2a\int(2+\cos 2\theta)\tan^2\theta\, d\theta$$

$\int\cos x\, dx = \sin x$.

$\int\cos 2x\, dx = \frac{\sin 2x}{2}$.

$$= -2a\int(1+2\cos^2\theta)\tan^2\theta\, d\theta = -2a\left[\int\tan^2\theta\, d\theta + \int 2\sin^2\theta\, d\theta\right]$$



$$= -2a\left[\int(\sec^2\theta - 1)\, d\theta + \int(1-\cos 2\theta)\, d\theta\right] = -2a\left[\tan\theta - \cancel{\theta} + \cancel{\theta} - \frac{\sin 2\theta}{2}\right] + c$$

$$= a\sin 2\theta - 2a\tan\theta + c = \cancel{a}\frac{\sqrt{a^2-x^2}}{\cancel{a}} - 2a\sqrt{\frac{1-x/a}{1+x/a}} + c.$$

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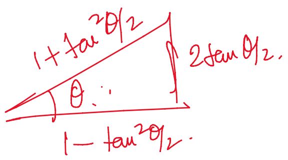
$$I = \int\frac{f(\sin x/\cos x)}{g(\sin x)\,h(\cos x)}\,dx. \rightarrow \text{use } \tan x/2 \rightarrow \quad \sin x = \frac{2\tan x/2}{1+\tan^2 x/2}.$$

$$\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$$

$$\int \phi(\tan x/2)\,dx. \qquad t = \tan x/2.$$



$x^y + y^x = c$ find $\frac{dy}{dx}$

$\log(a+b)$ ✗.

$\log(a\times b) = \log a + \log b$

$u = x^y$ $\qquad v = y^x$

$\log u = y\log x$. $\qquad \log v = x\log y$.

$\frac{1}{u}\cdot\frac{du}{dx} = \frac{dy}{dx}\log x + \frac{1}{x}y.$ $\qquad \frac{1}{v}\frac{dv}{dx} = \log y + x\frac{1}{y}\frac{dy}{dx}.$

$\frac{du}{dx} = x^y\left[\log x\frac{dy}{dx} + \frac{y}{x}\right]$ $\qquad \frac{dv}{dx} = y^x\left[\log y + \frac{x}{y}\frac{dy}{dx}\right]$

$u + v = c.$

$$u+v=c.$$

$$\frac{d}{dx}(u+v)=\frac{d}{dx}(c) \qquad \frac{du}{dx}+\frac{dv}{dx}=0.$$

$$x^y\log x\frac{dy}{dx}+x^{y-1}y+y^x\log y+y^{x-1}x\frac{dy}{dx}=0.$$

$$\left(x^y\log x+xy^{x-1}\right)\frac{dy}{dx}=-\left(x^{y-1}y+y^x\log y\right)$$

$$\frac{dy}{dx}=-\frac{\left(x^{y-1}y+y^x\log y\right)}{x^y\log x+xy^{x-1}}$$

$$y=e^x f(x) \qquad \frac{dy}{dx}=e^x f(x)+e^x f'(x)=e^x[f(x)+f'(x)]$$

$$y=\boxed{\int e^x[f(x)+f'(x)]\,dx.}$$

$$I=\int e^x(\sin x+\cos x)\,dx \quad =e^x\sin x.$$

$$I=\int e^t\left(\frac{1}{t^2}-\frac{1}{t}\right)dt.=e^t\left(-\frac{1}{t}\right)=-\frac{e^t}{t}.$$

$$f(t)=\frac{1}{t^2} \quad f'(t)=\frac{-2}{t^3}. \qquad f(t)=-\frac{1}{t}\longrightarrow f'(t)=\frac{1}{t^2}.$$

$$y=\int\frac{dx}{\sqrt{(x-a)(x-b)}}\longrightarrow \text{quadratic.}\rightarrow ax^2+bx+c$$

$$\left(x+\frac{b}{2a}\right)^2-\left(\frac{\sqrt{b^2-4ac}}{2a}\right)^2.$$

$$x+\frac{b}{2a}=t$$

$$t^2-k^2.$$

$$t=k\sec\theta.$$

$$dt=k\sec\theta\tan\theta\,d\theta.$$

$$\sqrt{t^2-k^2}=k\tan\theta.$$

$$\int\sec\theta\,d\theta.$$

$$\downarrow$$

$$=\log|\sec\theta+\tan\theta|$$

$$\frac{d}{d\theta}(\sec\theta+\tan\theta)=\sec\theta\tan\theta+\sec^2\theta=\sec\theta(\tan\theta+\sec\theta)$$

$$\int\sec\theta\,d\theta=\int\frac{d(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta.}=\log|\sec\theta+\tan\theta|$$

$$I = \int \frac{\cos x \, dx}{\sqrt{(\sin x + a)(\sin x + b)}}$$