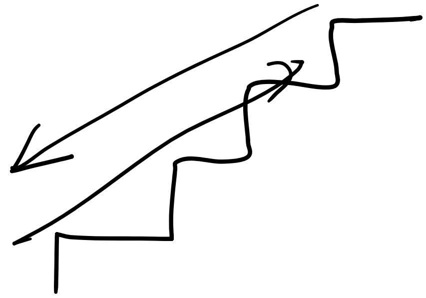
AP + GP ..



Sequences

Infinites

Progression

1, 4, 8, 16 ---.

1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ ---.

AGP

1+2+3+...+n

1^4+2^4

1^5+2^5

?? $1^{10}+2^{10}$

2, 4, 6, 8 ----

2 2

GP

2, 4, 8, 16 ---

×2 ×2

$y = \boxed{2x+3}$

$x \in N$

1, 2, 3, 4 .... natural numbers

AP

$y_1 = 2.1+3 = 5$

$y_2 = 2.2+3 = 7$

$y_3 = 2.3+3 = 9$

$y_4 = 2.4+3 = 11$ ...

Change of origin & change of Scale ..

+ −

$y = x \pm 6$

Scale change ÷ ×

which changes factors

AP

If the variable is same

If the variable is same

GP > AP

2, 4, 8, 16 — $r = 2$

2, 4, 6, 8 — $d = 2$

AP? $d = 100$ — 1, 100, 200

GP => $r => 2$ — 1, 2, 4

Iff → the parameters are same ..

4, 4, 4, 4 ___ ___ . Is it a progression?

$r = 1$ $d = 0$

Constant progression ...

Infinite series ..

$-\infty$, 1 ---- ∞

1, 2 -- ∞

-1, -2 --- $-\infty$

either lower bound or upper bound or both are unknown ..

UB / LB / Both

9062395723

fibonacci series

$\boxed{5, 7,}$ 12, 19, 31, ...

$a_n = a_{n-1} + a_{n-2}$

$S_n = 2n^2 + 3n$

+ + + + - - -   - - - -

finding the first negative number..

20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$ - - - - . .

$(20 - 19\frac{1}{4})$
$= 20 - 19.75 = 3/4$

In 27th > 0

$\boxed{d = -3/4}$

$t_n < 0$

28th term
must be < 0.

$20 + (28-1)(-\frac{3}{4})$
$= 20 - \frac{27\times 3}{4}$
$= 20 - 81/4$
$= 20 - 20.25$
$= -0.25$

$n > 27$
$n = 28$

$n > 27.99$
$n = 28$

$a + (n-1)d < 0$
$20 + (n-1)(-3/4) < 0$
$80 - 3n + 3 < 0$
$n > 83/3$
$n > 27\frac{2}{3}$
$\cancel{28} \boxed{n \to 28}$

the 28 ~~th term~~
⇒ the first negative term..

#

Nth term from end

1st term from end = last term
$= t'_1 = l - (1-1)d$
$= l$

2nd " " " $\Rightarrow l - (2-1)d$

$t'_n = l - (n-1)d$

$t_n + t'_n = a + l$

$(t_{n+1} - t_n) = d = \left(\frac{l-a}{n+1}\right)$

2011/2
(CMI) Modulus Problems

CMI 2009

Modulus Problems

$|x-1|, \; 3, \; |x-3|$ 3 term of AP Imp

Then find 6th term of the AP.

Case I $\quad x<1 \quad |x-1| = -(x-1)$

$|x-3| = -(x-3)$

$2b = a+c$

$1-x, \; 3, \; 3-x$ AP $\quad 6 = 1-x+3-x$

$x = -1$

$2, 3, 4 \quad 6^{th} \to 7$

Case II $\quad 1<x<3 \quad |x-1| = x-1$

$|x-3| = -(x-3) = 3-x$

$6 = \cancel{x} - 1 + 3 - \cancel{x}$

$6 \neq 2$

Case III $\quad x>3 \quad |x-1| = x-1$

$|x-3| = x-3$

$6 = x-1+x-3$

$x = 5$

$\boxed{4, 3, 2}$

$a_1, a_2 \dots a_n$ AP $\quad d$

$a_1k, a_2k, \dots a_nk$

$\frac{a_1}{k}, \frac{a_2}{k} \dots \frac{a_n}{k}$ in AP? Yes for all

d Remains same in all cases.

$kd \quad d/k \quad$ But not I Case

$a_2k - a_1k$

$=(a_2-a_1)k$

$a_2k - a_1k$

$= (a_2 - a_1)k$

$= dk$

$S = \frac{n}{2}\left[2a + (n-1)d\right]$

GP using Problem Sol^n

1, 22, 4444, 88888888, - - - -

find 1025th term??

$2^9 / 2^{10} / 2^{11} / 2^{12}$

digits → 1, 2, 4, 8 . . . . ; 2^{n-1}

$1 + 2 + 4 + \cdots 8 + \cdots 2^{n-1} < 1025 \le 1 + 2 + 4 + 8 + \cdots + 2^n$

$\Rightarrow \frac{(2-1)(1 + 2 + 2^2 + 2^3 + \cdots + 2^{n-1})}{(2-1)} < 1025 \le \frac{(2-1)(1 + 2 + 2^2 + 2^3 + \cdots 2^n)}{(2-1)}$

$2^n - 1 < 1025 \le 2^{n+1} - 1$

$2^n < 1026 \le 2^{n+1}$

$2^{10} = 1024$

$2^{n+1} \geq 1026 > 2^n$

$2^{n+1} \geq 1026 > 1024$

$2^{n+1} > 1024$

$2^{n+1} > 2^{10}$

$n + 1 > 10$

$\boxed{n > 9}$

$\boxed{n = \text{with term}}$

$2^{10} \to$

Recurring Decimal type Problems..

Recurring Decimal type problems..

$0.3\dot{2}\dot{7}$

$= 0.327272727$

$= 0.3 + 0.027 + 0.00027 + 0.0000027 + \ldots \infty$

$= \frac{3}{10} + \frac{27}{10^3} + \frac{27}{10^5} + \frac{27}{10^7} + \ldots \infty$

$= \frac{3}{10} + \frac{27}{10^3}\left(\frac{1}{1 - 1/10^2}\right)$

$\frac{1}{1-\frac{1}{100}} \Rightarrow \frac{1}{\frac{99}{100}} \Rightarrow \frac{100}{99}$

$\frac{a}{1-r}$

$= 3/10 + \frac{27}{990}$

$= \frac{297+27}{990} = \frac{324}{990}$

Universal Process for Recurring Decimal

P → not Recurring

p → p in number

Q → the Recurring Period of q figures.

R = value of the decimal

$R = 0.PQQQ \ldots$

$10^p \times R = P.QQQ \ldots$

$10^{p+q} \times R = PQ.QQQ \ldots$

Here, $R = \frac{PQ - P}{(10^{p+q} - 10^p)}$

Here, $R = \dfrac{P\varnothing - I}{(10^{p+q} - 10^{p})}$

$R = 0.\dot{3}$ $\qquad R = \dfrac{3}{10-1} = 3/9 = 1/3$

Here $R = 0.32\dot{5}\dot{8}\ldots$ — (i)

$R = 0.32\,58\,58\,58\,58\ldots$

$10^{2} \times R = 32.585858\ldots$ (ii)

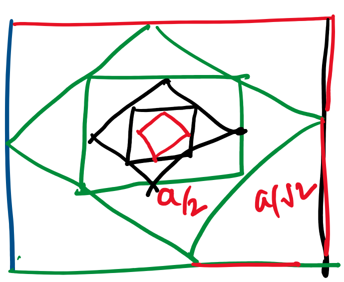
$10^{4} \times R = 3258.585858\ldots$ (iii)

(iii) − (ii) →

$$\begin{array}{r} 3258.5858\ldots \\ 32.5858\ldots \\ \hline 3226.0000 \end{array}$$

$9900R = 3226$

$R = \dfrac{3226}{9900} = \dfrac{1613}{4950}$



→ a^2 for outermost square

$\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$

$a,\ a/\sqrt{2},\ \dfrac{a}{2},\ \dfrac{a}{2\sqrt{2}},\ \dfrac{a}{4},$

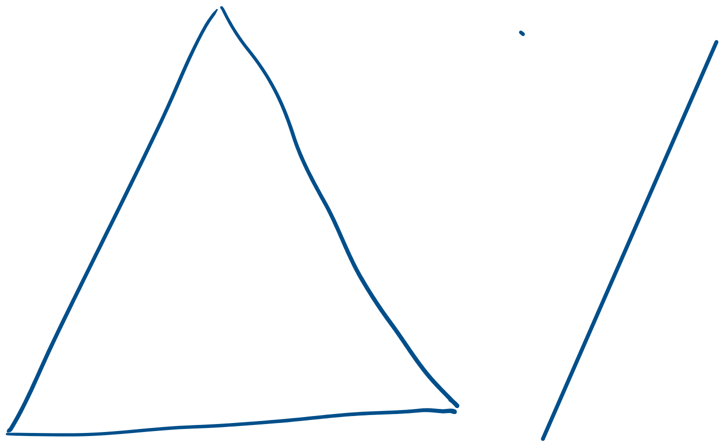
Sum of Areas $\quad (a)^2 + \left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2 + \ldots$

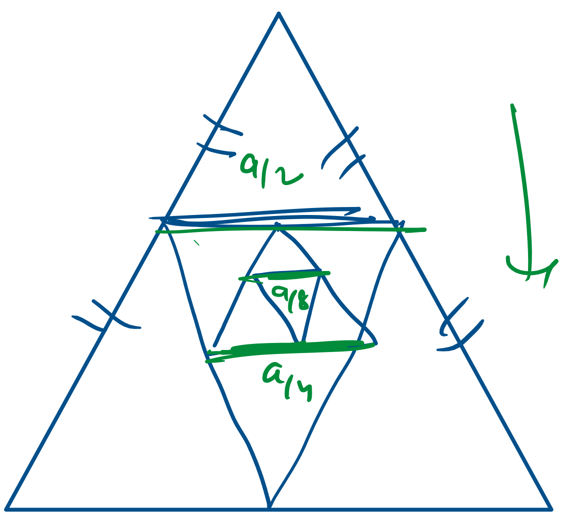
$= a^2\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right)$

#

$$= a^2\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)$$

$$= a^2\left(\cancel{1+1/2}\ \frac{1}{1-1/2}\right) = 2a^2$$





$a,\ a/2,\ a/4,\ a/8$

$S = 3(a + a/2 + a/4 + \cdots)$

$= 3 \cdot \left(\frac{a}{1-1/2}\right) \to \infty$ GP

$= 6a$

$\frac{a/2}{a} = \frac{1}{2}$

$\frac{a/4}{a/2} = 1/2$

a 3

A → H

AM = $\sum \frac{a}{n}$ GP = $(x_1 x_2 \cdots x_n)^{1/n}$

HM = $\frac{n}{\sum \frac{1}{x_i}}$

A G H a, b, c

number are

then the eq which can have, a, b, c as their Roots

$$X^3 - 3AX^2 + \frac{3G^3}{H}x - G^3 = 0$$

In general if $a_1, a_2, \ldots a_n$ if any of them is unequal to the others then

$$\boxed{AM > GM > HM}$$

$$\boxed{\frac{AM}{GM} = \frac{GM}{HM}}$$

$$(a-b)^2 \geq 0$$

$$a^2 - 2ab + b^2 \geq 0$$

$$\boxed{G^2 = AH}$$

AGS

$$a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \ldots$$

$$[a+(n-1)d]\, r^{n-1}$$

t_n

$$S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]\, r^n}{(1-r)}$$

$$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$$

$$1 + 4\left(\frac{1}{5}\right) + 7\left(\frac{1}{5}\right)^2 + 10\left(\frac{1}{5^3}\right) + \ldots$$

AP $1, 4, 7, 10 \ldots$

GP $1, \frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3} \ldots$

$$t_n = [1+(n-1)3] \times 1 \cdot \left(\frac{1}{5}\right)^{n-1}$$

$$= \boxed{(3n-2)\left(\frac{1}{5}\right)^{n-1}}$$

$t_n = ($

$= \boxed{(3n-2)\left(\frac{1}{5}\right)^{n-1}}$

$S_n = ??$