AP, GP, HP Series

AP: $\overset{T_1}{a}, \overset{T_2}{a+d}, \overset{T_3}{a+2d}, \ldots\ldots$ [common difference = d]

$a \xrightarrow{+d} a+d \xrightarrow{+d} a+2d$

First term = a.

common diff = d.

$$T_n = a + (n-1)d.$$

S_n = sum of 'n' terms of AP.

$$= a + [a+d] + [a+2d] + \cdots + [a+(n-1)d]$$

$$= \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[T_1 + T_n].$$

GP: $\overset{T_1}{a}, \overset{T_2}{ar}, \overset{T_3}{ar^2}, \ldots\ldots$ [r = common ratio]

$a \xrightarrow{\times r} ar \xrightarrow{\times r} ar^2$

First term = a

common Ratio = r.

$$T_n = ar^{n-1}$$

$$S_n = a + (ar) + (ar^2) + \cdots + (ar^{n-1})$$

$$= \begin{cases} \dfrac{a(r^n-1)}{(r-1)}, & r > 1. \\ \dfrac{a(1-r^n)}{(1-r)}, & r < 1 \end{cases}$$

Infinite GP: $S_\infty = \dfrac{a}{1-r}, \quad r < 1.$

HP: Eg: 3 no.s a, b, c in HP $\Rightarrow$ $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

Eg: Suppose 3 no.s a, b, c are in AP.

$\Rightarrow$ There is a common diff them $\Rightarrow d = b - a = c - b$.

$a \pm k, \; b \pm k, \; c \pm k \Rightarrow AP$.

$a \cdot k, \; b \cdot k, \; c \cdot k \Rightarrow AP \; (k \neq 0)$

$\frac{a}{k}, \; \frac{b}{k}, \; \frac{c}{k} \Rightarrow AP \; (k \neq 0)$.

Eg: Suppose 3 no.s a, b, c are in GP.

$\Rightarrow$ There is a common ratio $\Rightarrow r = \frac{b}{a} = \frac{c}{b}$.

$a \pm k, \; b \pm k, \; c \pm k \Rightarrow$ May/may not be in GP.

$a \cdot k, \; b \cdot k, \; c \cdot k \Rightarrow GP \; (k \neq 0) \; [cr = r]$.

$\frac{a}{k}, \; \frac{b}{k}, \; \frac{c}{k} \Rightarrow GP \; (k \neq 0) \; [cr = r]$.

Q. Consider a seq. $\{a_n\}$ where $a_n = 2n^2 + 3$. Check if the seq is in AP.

$n = 1 \Rightarrow a_1 = 5$

$n = 2 \Rightarrow a_2 = 11 \; \big\} +6$ Not in AP.

$n = 3 \Rightarrow a_3 = 21 \; \big\} +10$

$n = 4 \Rightarrow a_4 = 35 \; \big\} +14$

Q. The sum of 'n' terms of 2 AP's are in ratio $(2n+1):(2n-1)$. Find the ratio of the 10th terms.

$AP_1 : a_1, d_1$

$AP_2 : a_2, d_2$.

$$\frac{S_n^1}{S_n^2} = \frac{\cancel{\frac{n}{2}}[2a_1 + (n-1)d_1]}{\cancel{\frac{n}{2}}[2a_2 + (n-1)d_2]} = \frac{2n+1}{2n-1}$$

Reqd, $\frac{T_{10}^1}{T_{10}^2} = \frac{a_1 + 9d_1}{a_2 + 9d_2}$

$$\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{2n+1}{2n-1}$$

$= \frac{39}{37}$.

$\frac{n-1}{2} = 9 \Rightarrow n = 19$.

$= \frac{39}{37}$; $\frac{n-1}{2} = 9 \Rightarrow n = 19$.

Q. If the sum of 'p' terms in AP is q, & the sum of 'q' terms in AP is 'p'. Find the sum of $(p+q)$ terms and $(p-q)$ terms, $(p>q)$

$$\begin{cases} S_p = \frac{p}{2}[2a + (p-1)d] = q \\ S_q = \frac{q}{2}[2a + (q-1)d] = p \end{cases}$$

Solve for a, d.

$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$S_{p-q} = \frac{p-q}{2}[2a + (p-q-1)d]$

$2a + (p-1)d = \frac{2q}{p}$ --- (i)

$2a + (q-1)d = \frac{2p}{q}$ --- (ii)

$(p-1-q+1)d = 2\left(\frac{q}{p} - \frac{p}{q}\right)$

$(p-q)d = 2\left(\frac{q^2-p^2}{pq}\right) = -2\frac{(p+q)(p-q)}{pq}$

$$\boxed{d = \frac{-2(p+q)}{pq}}$$

(i) $2a + (p-1)d = \frac{2q}{p}$

$2a = \frac{2q}{p} - (p-1)d$

$S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$= \frac{p+q}{2}\left[\frac{2q}{p} - (p-1)d + (p+q-1)d\right]$

$$= \frac{p+q}{2}\left[\frac{2q}{p} - (p-1)d + (p+q-1)d\right]$$

$$= \frac{p+q}{2}\left[\frac{2q}{p} - (\cancel{p}-1)d + (\cancel{p-1})d + qd\right]$$

$$= \frac{p+q}{2}\left[\frac{2q}{p} + \frac{q(-2)(p+q)}{pq}\right].$$

$$= (p+q)\left[\frac{q}{p} - \frac{q(p+q)}{pq}\right] = -(p+q)$$

$$S_{p-q} = (p-q)\left(\frac{2q}{p}+1\right) \quad [HW!].$$

Q. If $a^2(b+c)$, $b^2(c+a)$, $c^2(a+b)$ are in AP, check if a, b, c are also in AP.

$$2b^2(c+a) = a^2(b+c) + c^2(a+b)$$

$$2b^2(c+a) = \underline{a^2b} + a^2c + c^2a + c^2\underline{b}.$$

$$2b^2(c+a) = b(a^2+c^2) + ac(a+c).$$

$$2b^2(c+a) - ac(a+c) = b(a^2+c^2)$$

$$(a+c)[2b^2 - ac] = b[(a+c)^2 - 2ac]$$

AP: $T_2 - T_1 = T_3 - T_2$

$$b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(c+a)$$

$$b^2c + \underline{b^2a} - \underline{a^2b} - a^2c = c^2a + \underline{c^2b} - \underline{b^2c} - b^2a.$$

$$c(b^2-a^2) + ab(b-a) = bc(c-b) + a(c^2-b^2)$$

$$(b-a)[ab+bc+ca] = (c-b)[ab+bc+ca]$$

$$[ab+bc+ca](b-a-c+b) = 0.$$

$$(ab+bc+ca)(2b-a-c)=0$$

Either $ab+bc+ca=0$ or $2b-a-c=0$.

$$\boxed{2b=a+c}$$

→ a, b, c are in AP.