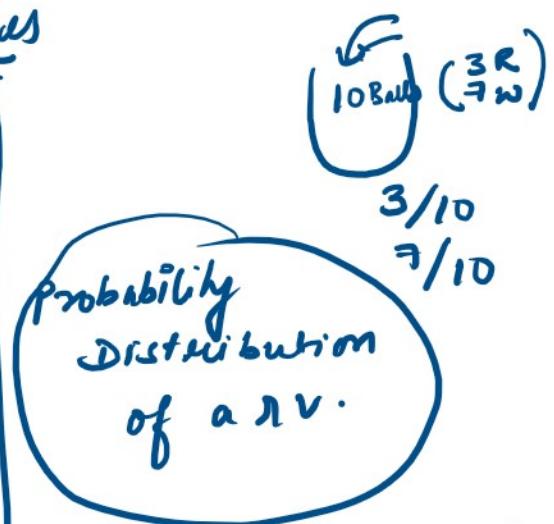


(n no. of
Random
variables)

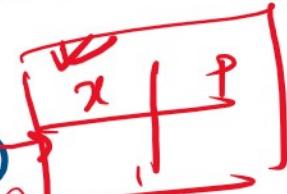
x_i	$P(X=x_i)$
x_1	P_1
x_2	P_2
x_3	P_3
\vdots	\vdots
x_n	P_m



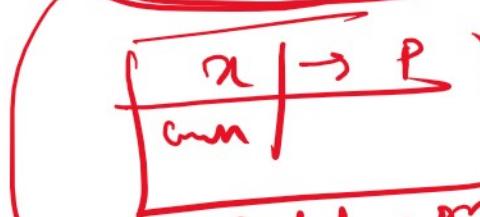
Random variables

Discrete R.V

Continuous R.V



p.m.f
probability mass function



p.d.f \rightarrow probability density function.

Expectation/Mean or Average
 $E(x)$

and Variance $V(x)$

x is a discrete r.v

$$\textcircled{1} E(x) = \sum_{x=0}^{\infty} x \cdot P(X=x)$$

$$\textcircled{2} x = c \quad E(x) = c$$

\textcircled{3}

$$x = c y$$

$$E(x) = c E(y)$$

x	P	xP
x_1	P_1	$x_1 P_1$
x_2	P_2	$x_2 P_2$
\vdots	\vdots	\vdots
x_m	P_m	$x_m P_m$

$$\sum xP = E(x)$$

$$\textcircled{4} x = a + b y$$

$$E(x) = a + b E(y)$$

(4) $x = a + b y$
 $E(x) = a + b E(y)$

(5) Variance $V(x) = E(x - E(x))^2$
or $E(x^2) - \{E(x)\}^2$
↓
mean.

(a) $x = c y$
 $V(x) = 0$

(b) $x = c y$
 $V(x) = c^2 V(y)$

(c) $x = a + b y$
 $V(x) = b^2 V(y)$

$V(x) = E(x^2) - E(x)^2 = \sigma_x^2$
 $\text{SD}(x) = \sqrt{V(x)}$

Q. If a fair coin is tossed twice, The no. of heads obtained (x) will have the following probability distribution

value of (x)	0	1	2	
$P(x=x)$	$1/4$	$1/2$	$1/4$	$\sum p = 1$

Calculate the expected value and variance of x .

Soln

$$E(x) = \sum x P(x=x)$$

$$= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} = 0 + 1 = 1$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= 0^2 - 1^2$$

$$V(x) = E(x^2) - \bar{x}^2$$

$$= E(x^2) - 1^2$$

$$E(x^2) = \sum x^2 P(x=x)$$

$$= 0^2 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4}$$

$$= 0 + \frac{1}{2} + 1 = \frac{3}{2}$$

$$V(x) = \frac{3}{2} - 1 = \frac{1}{2} \text{ (ans)}.$$

②	value of x	0	1	2	3
	$P(x=x)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{15}{28}$	$\frac{5}{28}$

Calculate $E(x)$ and $V(x)$.

$$E(x) = \sum x P(x)$$

$$= 0 \times \frac{1}{56} + 1 \times \frac{1}{56} + 2 \times \frac{1}{28} + 3 \times \frac{1}{28}$$

$$= \frac{1}{56} + \frac{5}{28} = \frac{105}{56}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \left(\frac{15}{8}\right)^2$$

$$E(x) = \frac{15}{8}$$

$$E(x^2) = \sum x^2 P(x=x) = 0^2 \times \frac{1}{56} + 1^2 \times \frac{1}{56} + 2^2 \times \frac{1}{28}$$

$$E(x^2) = \sum x^2 P(x=x) = 0^2 \times \frac{1}{56} + 1^2 \times \frac{1}{56} + 2^2 \times \frac{1}{28} + 3^2 \times \frac{1}{28}$$

$$= 0 + \frac{1}{56} + \frac{4}{28} + \frac{9}{28}$$

$$= \frac{1}{56} + \frac{13}{28}$$

$$= \frac{225}{56} \checkmark$$

$$\begin{aligned} V(x) &= E(x^2) - \left(\frac{15}{8}\right)^2 \\ &= \frac{225}{56} - \frac{225}{64} \\ &= 225 \left[\frac{1}{56} - \frac{1}{64} \right] \\ &= \underline{\underline{\frac{225}{448}}} \quad (\text{ans}) \end{aligned}$$

③ If $E(x) = 3$, $E(y) = 5$, Then $E(3x - 5y + 16) = ?$

$$\begin{aligned} E(3x - 5y + 16) &= 3E(x) - 5E(y) + 16 \\ &= (3 \times 3) - (5 \times 5) + 16 \\ &= 9 - 25 + 16 \\ &= 25 - 25 = 0 \quad (\text{ans}) \end{aligned}$$

$x = c$
$E(x) = c$
$x = 1c$
$E(x) = 1c$

④ If $E(x) = 7$ $E(y) = 10$
Then $E(3x + 4y + 20) = ?$

$$E(3x + 4y + 20) = \dots + 20$$

$$\begin{aligned}
 & E(x+y) \\
 &= 3E(x) + 4E(y) + 20 \\
 &= 3 \times 7 + 4 \times 10 + 20 \\
 &= 21 + 40 + 20 \\
 &= 81
 \end{aligned}$$

④ $E(x) = 4$ $v(x) = 9$ Then $E(x^2) = ?$

$$D_x = \sqrt{9} = 3$$

$$\begin{aligned}
 v(x) &= E(x^2) - [E(x)]^2 \\
 E(x^2) &= v(x) + \{E(x)\}^2 \\
 E(x^2) &= 9 + 4^2 \\
 &= 9 + 16
 \end{aligned}$$

$$E(x^2) = 25 \quad (\text{ans})$$

Joint Random variables (x and y) together.

$$\text{Cov}(x,y) = E[(x-E(x))(y-E(y))] \Rightarrow \text{Covariance of } x \text{ and } y.$$

$$\boxed{\text{Cov}(x,y) = E(xy) - E(x)E(y)}$$

Correlation coefficient, $r_{x,y} = \frac{\text{Cov}(x,y)}{\sqrt{v(x)} \sqrt{v(y)}}$

⑤ $E(x) = 2$ $E(y) = 1$ $E(xy) = 3$

$$E(x^2) = 5 \quad E(y^2) = 2$$

Calculate $\text{Cov}(x,y)$ and $r_{x,y}$.

$$\text{Cov}(x, y) = E(xy) - E(x)E(y)$$

$$= 3 - (2 \times 1) = 3 - 2 = 1 \checkmark$$

$$r_{xy} = \frac{\text{Cov}(x, y)}{\sqrt{v(x)} \sqrt{v(y)}} = \frac{1}{\sqrt{1}} = 1 \text{ (ane)}$$

$$v(x) = E(x^2) - E(x)^2$$

$$= 5 - 2^2$$

$$= 5 - 4$$

$$v(y) = E(y^2) - E(y)^2$$

$$= 2 - 1^2$$

$$v(x) = 1$$

$$\sqrt{v(x)} = 1$$

$$\begin{cases} = 2 - 1 \\ = 1 \end{cases} \therefore \delta_y = \sqrt{v(y)} = 1$$