03 March 2024 09:40 PM

Random variables

$x_i \rightarrow$

Rv(x)	P(X=x)
x_1 ✓	p_1
x_2 ✓ $\rightarrow$	p_2
x_3 ✓ $\rightarrow$	p_3
$\vdots$	$\vdots$
x_n ✓ $\rightarrow$	p_n

(n no. of Random variables

10 Balls $\left(\begin{matrix}3R\\7W\end{matrix}\right)$

3/10

7/10

probability Distribution of a r.v.

Random variables $\rightarrow$ Discrete R.V $\rightarrow$

x	p

p.m.f $\rightarrow$ probability mass function

Random variables $\rightarrow$ Continuous RV $\rightarrow$

x	$\rightarrow$ p
cum	

$\rightarrow$ p.d.f $\Rightarrow$ probability density function.

Expectation/Mean or Average and Variance $V(X)$

$E(X)$

X is a discrete r.v

① $E(X) = \sum_{x=0}^{\infty} x \cdot P(X=x)$

② $X = c$ ✓

$E(X) = c$ ✓

③ $X = cY$

$E(X) = cE(Y)$ ✓

x	p	xp
x_1	p_1	x_1p_1
x_2	p_2	x_2p_2
$\vdots$	$\vdots$	$\vdots$
x_n	p_n	x_np_n

$\Sigma xp = E(x)$

④ $X = a + bY$

$E(X) = a + bE(Y)$ ✓✓

(4) $x = a + by$

$E(x) = a + b\,E(y)$

(5) Variance $V(x) = E(x - E(x))^2$

or $E(x^2) - \underbrace{\{E(x)\}}_{\text{mean.}}{}^2$

(a) $x = c$

$\boxed{V(x) = 0}$

(b) $x = cy$

$\boxed{V(x) = c^2\,V(y)}$

(c) $x = a + by$

$\boxed{V(x) = b^2\,V(y)}$

$V(x) = E(x^2) - E(x)^2 = \sigma_x^2$

SD(x) Standard Deviation $(\sigma_x) = \sqrt{V(x)}$

Q. If a fair coin is tossed twice, the no. of heads obtained (x) will have the following probability distribution

value of (x)	0	1	2	
$P(X = x)$	1/4	1/2	1/4	$\Sigma P = 1$

Calculate the expected value and variance of X.

Soln $E(x) = \sum x\,P(X = x)$

$= 0 \times \frac{1}{4} + 1 \times \frac{1}{2} + \cancel{2} \times \frac{1}{\cancel{4}_2} = 0 + 1 = 1$

$V(x) = E(x^2) - (E(x))^2$

$= E(x^2) - 1^2$

$$= E(x^2) - 1^2$$

$$E(x^2) = \sum x^2 P(X=x)$$

$$= 0^2 \times \frac{1}{4} + 1^2 \times \frac{1}{2} + 2^2 \times \frac{1}{4}$$

$$= 0 + 1/2 + 1 = 3/2$$

$$V(x) = 3/2 - 1 = 1/2 \text{ (ans)}.$$

(2)

value of x	0	1	2	3
$P(X=x)$	$1/56$	$15/56$	$15/28$	$5/28$

Calculate $E(x)$ and $V(x)$.

$$E(x) = \sum x\ P(x)$$

$$= 0 \times 1/56 + 1 \times 1/56 + 2 \times 1/28 + 3 \times \frac{1}{28}$$

$$= \frac{1}{56} + \frac{5}{28} = \frac{105}{56}$$

$$E(x) = \frac{15}{8}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \left(\frac{15}{8}\right)^2$$

$$E(x^2) = \sum x^2\ P(X=x) = 0^2 \times \frac{1}{56} + 1^2 \times \frac{1}{56} + 2^2 \times \frac{1}{28}$$

$E(x^2) = \sum x^2 P(X=x) = 0^2 \times \frac{1}{56} + 1^2 \times \frac{1}{56} + 2 \times \frac{1}{28} + 3^2 \times \frac{1}{28}$

$= 0 + \frac{1}{56} + \frac{4}{28} + \frac{9}{28}$

$= \frac{1}{56} + \frac{13}{28}$

$= \frac{225}{56}$ ✓

$V(x) = E(x^2) - \left(\frac{15}{8}\right)^2$

$= \frac{225}{56} - \frac{225}{64}$

$= 225\left[\frac{1}{56} - \frac{1}{64}\right]$

$= \frac{\cancel{225}}{448}$ (ans)

(3) If $E(x) = 3$, $E(y) = 5$, Then $E(3x - 5y + 16) = ?$

$x = c$
$E(x) = c$

$x = kc$
$E(x) = kc$

$E(3x - 5y + 16)$

$= 3E(x) - 5E(y) + 16$

$= (3 \times 3) - (5 \times 5) + 16$

$= 9 - 25 + 16$

$= 25 - 25 = 0$ (ans).

(3) If $E(x) = 7$ $E(y) = 10$
then $E(3x + 4y + 20) = ?$

$E(3x + 4y + 20)$

$$E(3x + 4y + 20)$$

$$= 3E(x) + 4E(y) + 20$$

$$= 3 \times 7 + 4 \times 10 + 20$$

$$= 21 + 40 + 20$$

$$= 81$$

(9) $E(x) = 4$, $V(x) = 9$ then $E(x^2) = ?$

$\sigma_x = \sqrt{9} = 3$

$$V(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = V(x) + \{E(x)\}^2$$

$$E(x^2) = 9 + 4^2$$

$$= 9 + 16$$

$$E(x^2) = 25 \quad \text{(ans)}.$$

Joint Random variables (x and y) together.

$$Cov(x, y) = E\left[(x - E(x))(y - E(y))\right] \Rightarrow \text{Covariance of } x \text{ and } y.$$

$$\boxed{Cov(x, y) = E(xy) - E(x)E(y)}$$

Correlation coefficient, $r_{x,y} = \dfrac{cov(x, y)}{\sqrt{V(x)}\sqrt{V(y)}}$

Q $E(x) = 2$ $E(y) = 1$ $E(xy) = 3$

$E(x^2) = 5$ $E(y^2) = 2$

Calculate $Cov(x, y)$ and $r_{x,y}$.

$$\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$$
$$= 3 - (2 \times 1) = 3 - 2 = 1 \checkmark$$

$$r_{X,Y} = \frac{\text{Cov}(X,Y)}{\sqrt{V(X)}\,\sqrt{V(Y)}} = \frac{1}{1 \times 1} = 1 \text{ (ans)}$$

$$V(X) = E(X^2) - E(X)^2$$
$$= 5 - 2^2$$
$$= 5 - 4$$
$$\boxed{V(X) = 1}$$
$$\sqrt{V(X)} = 1$$

$$V(Y) = E(Y^2) - E(Y)^2$$
$$= 2 - 1^2$$
$$= 2 - 1$$
$$= 1$$
$$\therefore \sigma_Y = \sqrt{V(Y)} = 1$$