HYPOTHESIS 2 25.4.24

25 April 2024 18:38

Error → How Big is the error

Redefinition of the Error?

Type I

Type II

9062395123

Wrong person → entering into your life — A Type II

Right person → You never get to marry — B Type I

Power of a Test → 1 − Type II Error

Ho → I'm a good person

Ho True   Ho False

Accept Ho

Reject Ho

	T	F
A	X	II
R	I	X

Type II > Type I
Error   Error

Error Error

	T	F
A	X	II
R	I	X

Reject H_0

P/C $\int$ Bin

Question

$X_1, X_2 \ldots X_n \rightarrow N(\theta, 1)$ mean, S.D

p $\quad H_0 : \theta = 2.5 \qquad H_1 = \theta = 4$ $\quad \alpha_1, \beta_1$

q $\quad$ Test I : Reject H_0 $\quad x_1 > 4$ $\quad (p, q) \rightarrow \alpha_1 \beta_1$

r $\quad$ Test II : Reject H_0 $\quad x_1 > 3$ $\quad (p, r) \rightarrow \alpha_2 \beta_2$

$\alpha_1 = \alpha_2 \qquad \beta_1 = \beta_2$

Ans

$\boxed{\text{Test I}}$ $\qquad$ CR : $x > 4$

$$\alpha_1 = P(\text{Type I Error})$$
$$= P(\text{Reject } H_0 \mid H_0 \text{ is true})$$
$$= P(X_1 > 4 \mid \theta = 2.5)$$
$$= P\left(\frac{X_1 - \theta}{\sqrt{1}} > \frac{4 - \theta}{\sqrt{1}} \;\middle|\; \theta = 2.5\right)$$

$$= P\left(\frac{\quad}{\sqrt{1}} \quad \frac{\quad}{\sqrt{1}}\right)$$

$$= P\left(Z > \frac{4 - 2.5}{\sqrt{1}}\right)$$

$$= P(Z > 1.5)$$

$$= 1 - (Z \le 1.5)$$

$$= 1 - 0.933$$

$$\alpha_1 = 0.067$$

Type II Error II

$$\beta_1 = P(\text{Type II Error})$$

$$= P(\text{Don't Accept } H_0 \mid H_1 \text{ is true})$$

$$= 1 - P(\text{Reject } H_0 \mid H_1 \text{ is true})$$

$$= 1 - P(X_1 > 4 \mid \theta = 4)$$

$$= 1 - P\left(\frac{X_1 - \theta}{\sqrt{1}} > \frac{4 - \theta}{\sqrt{1}} \,\Big|\, \theta = 4\right)$$

$N = (0, 1)$ mean, Variance

$$= 1 - P(Z > 0)$$

$$\beta_1 = \cancel{1 - P(} \; 0.5$$

Test II Error I $\rightarrow \alpha_2$

$$N(\theta, 1) \Rightarrow \bar{X} \sim N\left(\theta, \frac{1}{9}\right)$$

$$\alpha_2 = P(\text{Reject } H_0 \mid H_0 \text{ true})$$

$$(\bar{X} > 3 \mid \theta = 2.5)$$

$$\alpha_2 = P(\text{Rej } H_0 \mid H_0 \ldots)$$
$$= P(\bar{X} > 3 \mid \theta = 2.5)$$
$$= P\left(\frac{\bar{X} - 2.5}{\frac{1}{3}} > \frac{0.5}{1/3}\right)$$
$$= P(Z > 1.5)$$
$$= 1 - P(Z \le 1.5)$$
$$= \underline{0.067}$$

$$\beta_2 = 1 - P(\text{Rej } H_0 \mid H_1 \text{ is true})$$
$$= 1 - P(\bar{X} > 3 \mid \theta = 4)$$
$$= 1 - P\left[\frac{\bar{X} - 4}{1/3} \le \frac{-1}{1/3}\right]$$
$$= 1 - P(Z \ge -3)$$
$$= 1 - 0.99 = 0.01$$

$$z = \frac{\text{Actual} - \text{Expect}}{SE}$$

$\alpha_2 = 0.67$

$\beta_2 = 0.01$

$\alpha_1 = \alpha_2 = 0.67$

$\beta_1 > \beta_2$

θ x P(

x

$x > 4$

$x = 0, 1, 2, 3$

$x = 4, 5, 6, 7, 8, 9$

$$p(R) = \frac{6}{10} = 3/5$$

$1 - \frac{4}{\ldots}$ $2/5$

$x = \{0, 1, 2, \ldots$

$p(R) = \frac{}{10}$ $= 2/5$

$P(W) = \frac{4}{10}$

Type II — Finding an Unknown on the Basis of Given Error Calculations

PDF $f(x|\theta) = \theta/x \, (3/x)^\theta \quad x > 3$

$= 0 \quad o/w$

Based on X → the most powerful test for $\alpha = 0.1$ for testing $H_0: \theta = 1$, against $H_1: \theta = 2$

Rejects the hypothesis if $x < K$. $K = ?$

1 / 10/3 / 11/3 / 4

Ans: $f(x|\theta) = \frac{\theta}{x}\left(\frac{3}{x}\right)^\theta \quad x > 3$

CR: $x < K$

$H_0: \theta = 1$

$H_1: \theta = 2$

$\alpha = P(\text{Rej } H_0 \mid H_0 \text{ true})$

$= P(x < K \mid \theta = 1)$

$= \int_3^K \frac{3}{x^2} dx$

$= 3 \int_3^K \frac{1}{x^2} dx$

$=$

$= 3 \int x$

$= -\frac{3}{K} + \frac{3}{3 \cdot \frac{1}{3}} = 1 - \frac{3}{K}$

So, $0.1 = 1 - \frac{3}{K}$

$K = \frac{3}{0.9} = \frac{10}{3}$

ISI 2016 $f(x|H_0)$ PMF under NH

$X=x$	0	1	2	3
$f(x\|H_0)$	0.4	0.3	0.2	0.1
$f(x\|H_1)$	0.1	0.2	0.3	0.4

IPL RCB vs CSK

WPS BPS

8-1-7 9-4-5
9-2-7 14-8-6
14-7-7

Anju Bpana
Bijan Bima

$X \to H_0$
$X\ x \to H_1$

Reject H_0: $x > 3/2$

P(Rej H_0 | H_0 true)

$\alpha = P(\text{Rej } H_0 \mid H_0 \text{ true})$

$= P(x \geq 3/2 \mid H_0)$

$= P(x = 2 \mid H_0) + P(x = 3 \mid H_0)$

(PMF)

$0.2 + 0.1 = 0.3$

$=$

$\beta = P(x \geq 3/2 \mid H_1)$

$= P(x = 2 \mid H_1) + P(x = 3 \mid H_1)$

$= 0.3 + 0.4 = 0.7$

$\alpha < \beta$

Drivers (I) 500/6hr + 50 Rs/hr (First 6hr)

2nd (II) 450/5hr + 50 Rs/hr

2 hr 3 hr

$h > 5$ Indef (I) (II)

Uniform Distribution

$U(\theta, \theta^2)$ $\theta \in$ either $\{2, 3\}$

$H_0: \theta = 2$ $H_1: \theta = 3$

H_0 is Reject if $X \geq 3.5$

$\alpha, \beta = ?$

α = Size of the Test

β = Power of The Test

$$f(x|\theta) = \frac{1}{\theta^2 - \theta} \quad 0 < x < \theta^2$$

$H_0: \theta = 2$

$H_1: \theta = 3$

$\boxed{CR: X \geq 3.5}$

$$\alpha + \beta = P(x \geq 3.5 | \theta = 2) + P(x \geq 3.5 | \theta = 3)$$

$$= \int_{3.5}^{\theta^2} \frac{1}{\underbrace{\theta^2 - \theta}_{2}} dx + \int_{3.5}^{\theta^2} \frac{1}{\underbrace{\theta^2 - \theta}_{3}} dx$$

$$= \int_{3.5}^{4} \frac{1}{2} dx + \int_{3.5}^{9} \frac{1}{6} dx$$

$$= \boxed{1.167}$$