

Power of a Test →  $1 - \text{Type II Error}$

$H_0 \rightarrow$  I'm a good person       $H_0$  true       $H_0$  false

	Accept $H_0$	Reject $H_0$
<u>(A)</u>	X	<u>(IP)</u>
R	I	X

~~Type II Error~~ Type I Error

Reject  $H_0$

	$\bar{1}$	F
A	X	$\bar{1}$
R	I	X

Corr Err

P/c      J      bn

Number

$x_1, x_2, \dots, x_n \rightarrow N(\theta, \sigma^2)$  <sup>mean</sup>  $\rightarrow$  SD

$H_0: \theta = 2.5$        $H_1: \theta = 4$   
 Test 1: Reject  $H_0$  if  $x_1 > 4$   
 Test 2: Reject  $H_0$  if  $x_1 > 3$   
 $\alpha_1 = \alpha_2$        $\beta_1 = \beta_2$

$\alpha_1, \beta_1$   
 $(p, q) \rightarrow \alpha, \beta_1$   
 $(b, r) \rightarrow \alpha_2, \beta_2$

Answer

Test 1

CR:  $x > 4$

$$\begin{aligned}
 \alpha_1 &= P(\text{Type 2 Error}) \\
 &= P(\text{Reject } H_0 | H_0 \text{ true}) \\
 &= P(x_1 > 4 | \theta = 2.5) \\
 &= P\left(\frac{x_1 - \theta}{\sigma} > \frac{4 - \theta}{\sigma} \mid \theta = 2.5\right)
 \end{aligned}$$

$$\begin{aligned}
 &= P\left(\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > \frac{4 - 2.5}{\sqrt{1}}\right) \\
 &= P(Z > 1.5) \\
 &= 1 - P(Z \leq 1.5) \\
 &= 1 - 0.933 \\
 &\alpha_1 = \underline{0.067}
 \end{aligned}$$

Test II  
Error II

$$\begin{aligned}
 \beta_1 &= P(\text{Type II Error}) \\
 &= P(\text{Do not reject } H_0 \mid H_1 \text{ is true}) \\
 &= 1 - P(\text{Reject } H_0 \mid H_1 \text{ is true}) \\
 &= 1 - P(X_1 > 4 \mid \theta = 4) \\
 &= 1 - P\left(\frac{X_1 - \theta}{\sigma/\sqrt{n}} > \frac{4 - \theta}{\sigma/\sqrt{n}} \mid \theta = 4\right) \\
 &= 1 - P(Z > 0)
 \end{aligned}$$

$N = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$   
men  $\rightarrow$  men

$\beta_1 = \cancel{1 - P(Z > 0)} = \underline{0.5}$

Test II  
Error I  $\rightarrow$   $\alpha_2$

$$N(0, 1) \Rightarrow \bar{X} \sim N\left(\theta, \frac{1}{9}\right)$$

$$\alpha_2 = P(\text{Reject } H_0 \mid H_0 \text{ true})$$

$$\begin{aligned}
 \alpha_2 &= P(\bar{X} > 3 | \theta = 2.5) \\
 &= P\left(\frac{\bar{X} - 2.5}{\frac{1}{3}} > \frac{0.5}{\frac{1}{3}}\right) \\
 &= P(Z > 1.5) \\
 &= 1 - P(Z \leq 1.5) \\
 &= \underline{0.067} \checkmark
 \end{aligned}$$

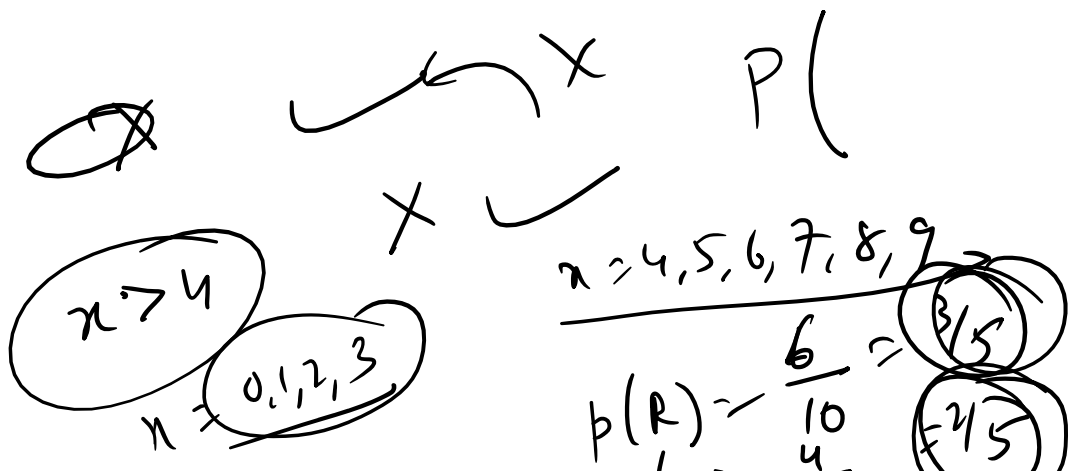
$$\begin{aligned}
 \beta_2 &= 1 - P(\bar{X} > 3 | \theta = 4) \\
 &= 1 - P\left(\frac{\bar{X} - 4}{\frac{1}{3}} \leq \frac{+1}{\frac{1}{3}}\right) \\
 &= 1 - P(Z \leq -3) \\
 &= 1 - 0.99 = 0.01
 \end{aligned}$$

$z = \frac{\text{Actual output} - \text{SE}}$

$1 - P(\beta_1 > \theta_0 | H_1, \text{is true})$

$$\begin{aligned}
 \alpha_2 &= 0.067 \\
 \beta_2 &= 0.01
 \end{aligned}$$

$$\begin{aligned}
 \alpha_1 &= \alpha_2 = 0.067 \\
 \beta_1 &> \beta_2
 \end{aligned}$$



$$n \sim (0, 1, 4)$$

$$P(R) = \frac{1}{10} \quad P(W) = \frac{4}{10} \quad \textcircled{2/5}$$

Type II

Finding an Upper Bound on the Basis of Given Error Calculations

PDF

$$f(x|\theta) = \frac{\theta}{x} \left(\frac{3}{x}\right)^\theta \quad x > 3$$

$$= 0 \quad \theta < 3$$

Based on  $X \rightarrow$  the most powerful test for  $d = 0.1$   
 for testing  $H_0: \theta = 1$ , against  $H_1: \theta = 2$   
 Rejects the hypothesis if  $x < K$ .  $K = ?$

~~$1/10, 3/4$~~

Ans:  $f(x|\theta) = \frac{\theta}{x} \left(\frac{3}{x}\right)^\theta \quad x > 3$

$H_0: \theta = 1$   
 $H_1: \theta = 2$

CR:  $x < K$

$$\alpha = P(\text{reject } H_0 | H_0 \text{ true})$$

$$= P(x < K | \theta = 1)$$

$$= \int_3^K \frac{3}{x^2} dx$$

$$= 3 \int_3^K \frac{1}{x^2} dx$$

$$= 3 \downarrow x$$

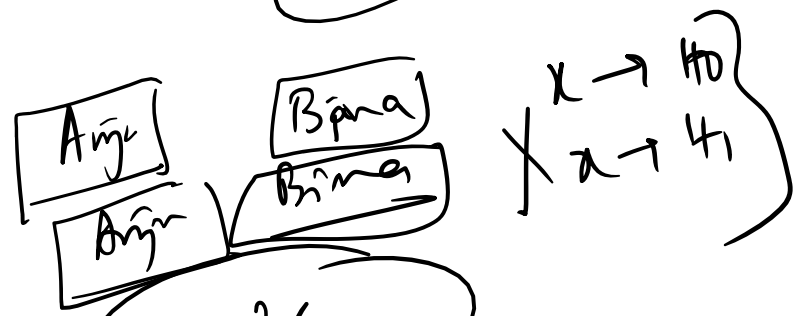
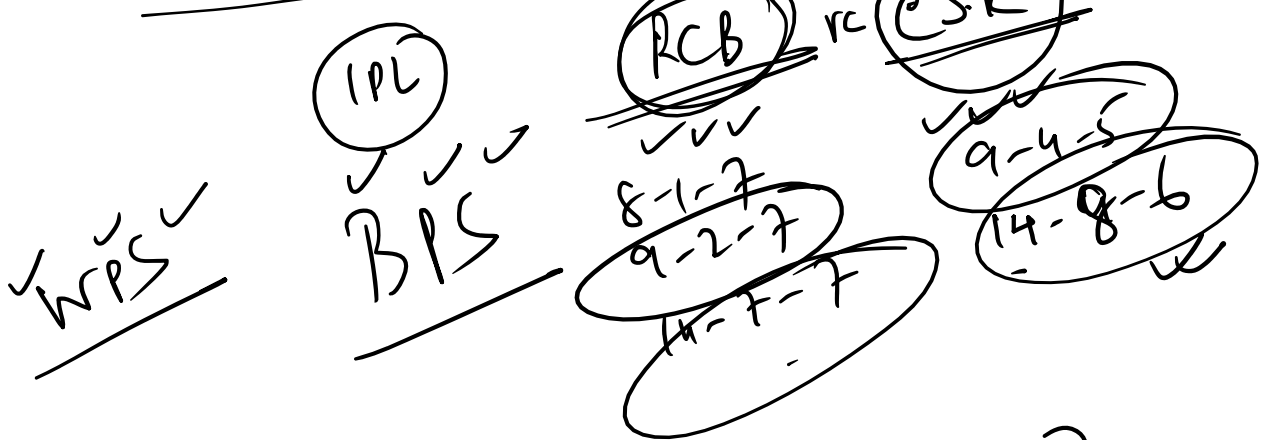
$$= -\frac{3}{k} + 3 \cdot \frac{1}{3} = 1 - \frac{3}{k}$$

So,  $0.1 = 1 - \frac{3}{k}$

$$k = \frac{3}{0.9} = \frac{10}{3}$$

ISI 2016  $f(x|H_0)$  PMF under  $H_0$

$X = a$	0	1	2	2
$f(a H_0)$	0.4	0.3	0.2	0.1
$f(a H_1)$	0.1	0.2	0.3	0.4



Ague  $H_0$ :  $x > 3/2$

✓ - D / (Amir  $H_0$  |  $H_0$  true)

$$\begin{aligned} \alpha &= P(\text{Buy } H_0 \mid H_0 \text{ true}) \\ &= P(x \geq 3/2 \mid H_0) \\ &= P(x=2 \mid H_0) + P(x=3 \mid H_0) \\ &\quad \text{(PMF)} \\ &= 0.2 + 0.1 + 0.3 \end{aligned}$$

$$\begin{aligned} \beta &= P(x \geq 3/2 \mid H_1) \\ &= P(x=2 \mid H_1) + P(x=3 \mid H_1) \\ &= 0.3 + 0.4 = 0.7 \end{aligned}$$

$$\alpha < \beta$$

Drivers (I)  $500 / \overset{\text{first}}{6 \text{ hr}} + 50 \text{ rs/hr}$   
 2nd (II)  $450 / \overset{\text{dr}}{5 \text{ hr}} + 50 \text{ rs/hr}$

$h > 5$  Indef (I) (II)

(2 hr) (3 hr)

Uniform Distribution

$$U(\theta, \theta^2)$$

$$H_0: \theta = 2$$

$$H_1: \theta = 3$$

$\theta \in \text{unif over } \{2, 3\}$

$\alpha = \text{Size of the Test}$   
 $\beta = \text{Power of the Test}$

It is known that  $x > 3.5$   
 $\alpha, \beta = ?$

$$f(x|\theta) = \frac{1}{\theta^2 - \theta}$$

$$0 < x < \theta^2$$

$$H_0: \theta = 2$$

$$H_1: \theta = 3$$

$$(R: x > 3.5)$$

$$\alpha + \beta = P(x > 3.5 | \theta = 2) + P(x > 3.5 | \theta = 3)$$

$$= \int_{3.5}^4 \frac{1}{\theta^2 - \theta} dx + \int_{3.5}^9 \frac{1}{\theta^2 - \theta} dx$$

$$= \int_{3.5}^4 \frac{1}{2} dx + \int_{3.5}^9 \frac{1}{6} dx$$

$$= \boxed{1.167}$$