Producers

Product Market (Supply side)

Factor Market (Demand side).

Decision: opt level of q

Decision: opt level of L, K.

Obj: π-max

For product mkt: $\pi = P.q - c(q)$ → cost fn derived from cost minimization

For factor mkt $\pi = P f(L,K) - wL - rK$

Deriving the factor demand curves:

Prodn fn: $q = f(L, K)$, $\frac{\partial q}{\partial L} > 0$, $\frac{\partial q}{\partial K} > 0$.

$\pi = P.f(L,K) - wL - rK$. [Obj: π-max by choosing L, K].

FOC: $\frac{\partial \pi}{\partial L} = 0 \Rightarrow P.\frac{\partial q}{\partial L} - w = 0 \Rightarrow MP_L = \frac{w}{P}$

$\frac{\partial \pi}{\partial K} = 0 \Rightarrow P.\frac{\partial q}{\partial K} - r = 0 \Rightarrow MP_K = \frac{r}{P}$

→ solve for L^*, K^*

Q. Prodn fn: $q = K^{\alpha} L^{1-\alpha} - \beta K + \theta L$; $0 < \alpha < 1$, $\beta, \theta > 0$.

(i) Show that $q(\cdot)$ satisfies CRS.

$$q = f(L, K) = K^{\alpha} L^{1-\alpha} - \beta K + \theta L.$$

$$f(\lambda L, \lambda K) = (\lambda K)^{\alpha} (\lambda L)^{1-\alpha} - \beta(\lambda K) + \theta(\lambda L)$$

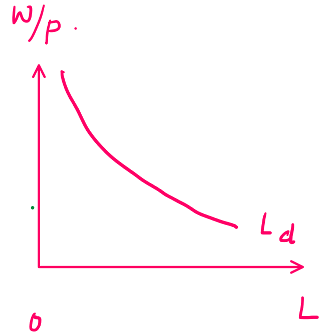
$$= \lambda [K^{\alpha} L^{1-\alpha} - \beta K + \theta L].$$

$$= \lambda^{1} \cdot f(L, K)$$

⇒ Homogeneous of degree 1 (CRS)

(ii) Verify if the labour demand curve is defined for all possible wage rates.

(ii) Verify if the labour demand curve is defined for all possible wage rates.



$$\pi = P \cdot f(L, K) - wL - rK$$

$$\pi = P\left[K^{\alpha}L^{1-\alpha} - \beta K + \theta L\right] - wL - rK$$

$$\frac{\partial \pi}{\partial L} = 0 \Rightarrow P\left[K^{\alpha}(1-\alpha)L^{-\alpha} + \theta\right] - w = 0$$

$$\Rightarrow K^{\alpha}(1-\alpha)L^{-\alpha} + \theta = \frac{w}{P}$$

$$\Rightarrow \left(\frac{K}{L}\right)^{\alpha} \cdot (1-\alpha) = \left(\frac{w}{P} - \theta\right)$$

$$\Rightarrow \left(\frac{L}{K}\right)^{-\alpha} = \frac{1}{(1-\alpha)} \cdot \left(\frac{w}{P} - \theta\right)$$

$$\Rightarrow \frac{L}{K} = \left\{\frac{1}{(1-\alpha)}\left(\frac{w}{P} - \theta\right)\right\}^{-1/\alpha}$$

$$\Rightarrow L = K\left[\frac{1}{(1-\alpha)}\underbrace{\left(\frac{w}{P} - \theta\right)}\right]^{-1/\alpha}$$

↳ Labour demand curve.

↳ valid only when $\left(\frac{w}{P} > \theta\right)$

(check)

(iii) Show that the demand function for capital is undefined when $r = 0$

$$\pi = P \cdot f(L, K) - wL - rK \qquad 0 < \alpha < 1$$

$$\pi = P\left[K^{\alpha}L^{1-\alpha} - \beta K + \theta L\right] - wL - rK$$

$$\frac{\partial \pi}{\partial K} = 0 \Rightarrow P\left[\alpha K^{\alpha-1}L^{1-\alpha} - \beta\right] - r = 0$$

$$\Rightarrow \alpha \cdot K^{\alpha-1}L^{1-\alpha} - \beta = \frac{r}{P}$$

$$\Rightarrow \alpha \cdot \left(\frac{L}{K}\right)^{1-\alpha} = \left(\frac{r}{P} + \beta\right)$$

$$\Rightarrow \left(\frac{L}{K}\right)^{1-\alpha} = \left(\frac{r}{P} + \beta\right)\frac{1}{\alpha}$$

$$\Rightarrow \frac{L}{K} = \left[\frac{1}{\alpha}\left(\frac{r}{P} + \beta\right)\right]^{1/1-\alpha}$$

$$\Rightarrow K = \frac{L}{\ldots}$$

$$\Rightarrow \quad K = \frac{L}{\left[\frac{1}{\alpha}\left(\frac{r}{P}+\beta\right)\right]^{1/1-\alpha}}$$

capital demand function

Put $r = 0 \Rightarrow K_0 = \frac{L}{\left[\frac{1}{\alpha}(0+\beta)\right]^{1/1-\alpha}} = \frac{L}{\left(\frac{\beta}{\alpha}\right)^{\frac{1}{1-\alpha}}}$

finite value.

When $r=0$, capital demand is defined.