30 August 2023 14:42

Matrix logic

Distinct numbers are arranged in an $m \times n$ rectangular table with m *rows* and n *columns* so that in each row the numbers are in increasing order (left to right), and in each column the numbers are in increasing order (top to bottom). Such a table is called a *sorted* table and each location of the table containing a number is called a *cell*. Two examples of sorted tables with 3 rows and 4 columns (and thus $3 \times 4 = 12$ cells) are shown below.

3	12	33	64
15	26	37	78
19	40	51	92

5	22	53	68
18	36	67	78
19	45	81	92

We index the cells of the table with a pair of integers (i, j), with the top-left corner being $(1, 1)$ and the bottom-right corner being (m, n). Observe that the smallest entry in a sorted table can only occur in cell $(1, 1)$; however, note that the second smallest entry can appear either in cell $(1, 2)$, as in the first example above, or in cell $(2, 1)$ as in the second example above.

(i) (a) Assuming that $m, n \geqslant 3$, where in an $m \times n$ sorted table can the third-smallest entry appear?

(b) For any $k \geqslant 4$ satisfying $m, n \geqslant k$, where in an $m \times n$ sorted table can the k^{th} smallest entry appear? Justify your answer.

(ii) Given an $m \times n$ sorted table, consider the problem of determining whether a particular number y appears in the table. Outline a procedure that inspects at most $m + n - 1$ cells in the table, and that correctly determines whether or not y appears in the table. Briefly justify why your procedure terminates correctly in no more than $m + n - 1$ steps.
[Hint: As the first step, consider inspecting the top-right cell.]

(iii) Consider an $m \times n$ table, say A, which might not be sorted; an example is shown below. Obtain table B from A by re-arranging the entries in each row so that they are in sorted order. Then obtain table C from B by re-arranging the entries in each column so that they are in sorted order. Fill in tables B and C here:

A:

33	92	46	24
25	26	37	8
49	40	81	22

$\rightarrow$ B:

$\rightarrow$ C:

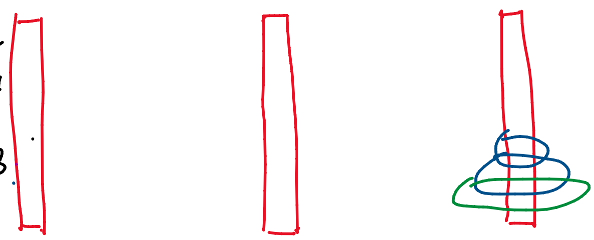
(iv) Show that for *any* $m \times n$ table A, performing the two operations from part (iii) results in a sorted table C.

	rings	moves
t_1	1	1
t_2	2	3
t_3	3	7
t_4	4	15

2, 4, 8

$t_n = ?$

t_{50}



A B C.

Transfer all the discs from pole A to pole C using the following conditions.

① you cannot move more than one disc at a time

② you can use pole B.

③ at no point in time ...

a) In how many moves can you achieve this? 15

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b) If there are 50 discs how many moves are needed.

③ at no point in time should a bigger disc be placed on a smaller disc.

$t_1 = 1$

$t_2 = 3$

$t_3 = 7$

$t_4 = 15$

$$\boxed{t_n = 2^n - 1}$$

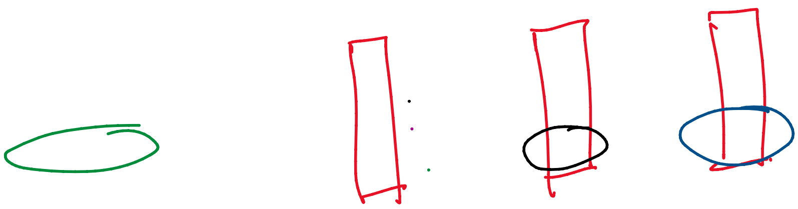
$$\cancel{t_2} - t_1 = 2^1$$
$$\cancel{t_3} - \cancel{t_2} = 2^2$$
$$\cancel{t_4} - \cancel{t_3} = 2^3$$
$$\vdots$$
$$t_n - \cancel{t_{n-1}} = 2^{n-1}$$

$$t_n - t_1 = 2^n - 2$$
$$t_n - 1 = 2^n - 2$$

$$S_n = a \cdot \frac{r^n - 1}{r - 1}$$

$$S_{n-1} = a \cdot \frac{r^{n-1} - 1}{r - 1}$$
$$= 2 \cdot \frac{2^{n-1} - 1}{2 - 1}$$
$$= 2(2^{n-1} - 1)$$
$$= 2^n - 2$$

ⓒ In a stack of 10 discs how many moves will be required to move the 7th disc from the top?



Tower of Hanoi

no of discs	Reqd move
1	$1 = 2^{1-1}$
2	$2 = 2^{2-1}$
3	$4 = 2^{3-1}$
4	$8 = 2^{4-1}$

7th disc → $2^{7-1} = 2^6 = 64$th move.

Row →

7	19	11	8	9
17	16	18	6	14
5	2	20	15	4
10	3	13	12	1

1–20 are placed at random in the boxes (no repetition)

In how many minimum moves can you arrange the nos in ascending order subject to the following conditions.

① you can swap 2 of them at a time

① you can swap 2 of them at a time
② While swapping row wise you can swap 2 even nos.
③ while swapping column wise you can swap a even with an odd.

C O N C A T E N A T E.

Arrange in ascending order by swapping.
What is the minimum no of swaps? and the maximum no of swaps.

Bubble sort

①.
②.
③.
④
⑤.

A A C C E E N | N O T | T.

A A C C E E N | N. O T | T.

Jail

□ □ □ □ □ □ □ □ □ □
1 2 3 4 5 6 7 8 9 10.

Assumption:
Intruder will act logically to free all the prisoners in the shortest time.

① He can open maximum 2 doors at a time.
② The doors opened must be in even-even or odd-odd pairs.
③ A sentry closes the first open door that he sees.
In 1 min the intruder can open 2 doors and in the next 1 min the sentry can close 1 door.
The prisoners can escape only when all doors are opened.
How long will it take to free the prisoners.
④ Both the intruder and the sentry takes 1 min to go from one door to the next.

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