ECO MATH 2.5.2024

02 May 2024 20:36

Mathematical Economics

Unused things SOC during

Hawkins Simon Conditions

Homogeneity / Homothetic functions

Multiplier finding

Lagrange Advanced Level

① Checking of homogeneity.

Type → Is the function homo. of degree (n)?

→ Checking the homogeneity.

$$f(x,y) = \frac{x^4 + 2x^2y^2}{x^3y + xy^4}$$

$$f(\lambda x, \lambda y) = \frac{(\lambda x)^4 + 2(\lambda x)^2(\lambda y)^2}{(\lambda x)^3(\lambda y) + (\lambda y)^4}$$

$$= \frac{\lambda^4}{\lambda^4} f(x,y) = \lambda^0 f(x,y)$$

Q.2

$$f(x,y) = \frac{3x^2y^2}{x^2+y^2}$$

9062395123

x^2+y^2

9062395123

$$= \lambda^2 \cdot x^2 \left(\frac{3x^2y^2}{x^2+y^2} \right)$$

$$= \lambda^2 f(x, y)$$

IFS 2014

Can degree of homogeneity < 0 ?

Yes

$$f(x,y) = \frac{x^2+y^2}{x^3+y^3}$$

$$= \frac{\lambda^2}{\lambda^3} f(x,y)$$

$$= \lambda^{-1} f(x,y)$$

Lagrangian Multiplier

max $U = 4xy$

Sub to $x + y \le 10$

$U = 4 \cdot 5 \cdot 5$

$U = 100$

$$L = 4xy + \lambda(10 - x - y)$$

$$\frac{\partial L}{\partial x} = 4y - \lambda = 0$$

$x = y$

$$\frac{\partial L}{\partial y} = 4x - \lambda = 0$$

$\frac{\partial L}{\partial y} =$ $10 - x - y = 0$ $x = y = 5$

$\frac{\partial L}{\partial \lambda} =$

Bordered Hessian det

$$\begin{vmatrix} L_{xx} & L_{xy} & L_{x\lambda} \\ L_{yx} & L_{yy} & L_{y\lambda} \\ L_{\lambda x} & L_{\lambda y} & L_{\lambda\lambda} \end{vmatrix} = \begin{vmatrix} 0 & 4 & -1 \\ 4 & 0 & -1 \\ -1 & -1 & 0 \end{vmatrix}$$

$= -4(-1) - 1(-4)$

$= 4 + 4 = 8 > 0$

Lagrange Multiplier with 2/3/more Conditions

$U = 4xy$

$2x + y \le 10$

$x \ge 4$

$y \le 3$

$$L = 4xy + \lambda_1(10 - 2x - y) + \lambda_2(4 - x) + \lambda_3(3 - y)$$

Elasticity Based Questions.

$$MR = P\left[1 - \frac{1}{|e_d|}\right]$$

Given

$$p = (12 - x)^{\frac{1}{2}}$$

$$0 \le x \le 12$$

$$\frac{dp}{dx} = \frac{1}{2}(12 - x)^{-\frac{1}{2}}(-1)$$

$$\frac{dx}{dp} = -2(12 - x)^{\frac{1}{2}}$$

$$|e_d| = \frac{-p}{x}\cdot\frac{dx}{dp} = -\frac{(12-x)^{\frac{1}{2}}}{x}\left[-2(12-x)^{\frac{1}{2}}\right]$$

$$= \frac{2(12-x)}{x}$$

$$R(x) = p \cdot x = (12-x)^{\frac{1}{2}} x$$

$$MR = \frac{dR}{dx} = \frac{1}{2}(12-x)^{-\frac{1}{2}}(-1)x + (12-x)^{\frac{1}{2}}$$

$$\Rightarrow \frac{(24 - x)}{2\sqrt{12 - x}}$$

RHS

$$P\left(1 - \frac{1}{|e_d|}\right)$$

$$= (12 - x)^{\frac{1}{2}}\left\{1 - \frac{x}{2(12-x)}\right\}$$

$$(12 - x)^{\frac{1}{2}}\left(\frac{24-3x}{2(12-x)}\right)$$

$= (12 \ldots$

$= (12-x)^{1/2} \left[\frac{24-3x}{2(12-x)} \right]$

$= \frac{24-3x}{2\sqrt{12-x}}$

Hence proved..

Q $aq + bp - k = 0$ $a, b, k > 0$

Find pt elasticity of dd when MR=0.

$p = -\frac{aq}{b} + \frac{k}{b}$

$R = PQ = -\frac{aq^2}{b} + \frac{kq}{b}$

$MR = -\frac{2aq}{b} + \frac{k}{b} = 0$

$\therefore \frac{2aq}{b} = \frac{k}{b}$

$q = \frac{k}{2a}$

Now, $\frac{dp}{dq} = -\frac{a}{b}$

$p = -\frac{ak}{2ab} + \frac{k}{b}$

$= \frac{-k+2k}{2b} = \frac{k}{2b}$

$|ed| = -\frac{dq}{dp} \cdot \frac{p}{q} = -\left[-\frac{b}{a} \cdot \frac{k/2b}{k/2a} \right]$

$= 1$

Important Relation

$$|e_d| = \frac{AR}{AR - MR}$$

Usage of integration

$TR = R(q)$

If $R(q=2) = 200$
$MR = 10 + 20q - 3q^2$

then find $TR = ?$

$\frac{d}{dq}(TR) = MR$

$\int MR\, dq = TR$

Ans:

$MR = \frac{dR}{dq} = 10 + 20q - 3q^2$

$\int dR = \int (10 + 20q - 3q^2)\, dq$

$TR(q) = 10q + \frac{20q^2}{2} - \frac{3q^3}{3} + K$

$TR(q) = 10q + 10q^2 - q^3 + K$

At $q = 2$

$10(2) + 10(4) - 8 + K = 200$

$K = 148$

$TR(q) = 10q + 10q^2 - q^3 + 148$

(7)

$$q = x_1^{1/2} x_2^{1/2}$$

$p_{x_2} = 4$

$p_{x_1} = 2$

Total Cost = (80)

Find max output subject to Cost Const..

$$C = r_1 x_1 + r_2 x_2$$

$$80 = 2x_1 + 4x_2$$

$$\boxed{\frac{r_1}{r_2} = \frac{1}{2}}$$

$$f_1 = \frac{\partial q}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$f_2 = \frac{\partial q}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

$$\boxed{\frac{f_1}{f_2} = \frac{x_2}{x_1}} \rightarrow$$

Condition for output maximisation

$$\boxed{\frac{f_1}{f_2} = \frac{r_1}{r_2}}$$

$$\frac{x_2}{x_1} = \frac{r_1}{r_2} = \frac{1}{2}$$

$$\boxed{x_1 = 2x_2}$$

Indo Cur

$80 = 2(2x_2) + 4x_2$

$x_2 = 10$ $x_1 = 20$

So, $q = (20)^{\frac{1}{2}} (10)^{\frac{1}{2}}$

$= \sqrt{200}$

MRTS finding from production function

$Q = 2\alpha KL - \beta L^2 - \gamma K^2$

$MRTS = \frac{MP_L}{MP_K}$ >1, <1, $=1$

$MP_L > MP_K$

$\frac{\partial Q}{\partial L} = 2\alpha K - 2\beta L$

$\frac{\partial Q}{\partial K} = 2\alpha L - 2\gamma K$

$MRTS = -\frac{dK}{dL} = \frac{\partial Q}{\ }$

$= \frac{2(\alpha K - \beta L)}{2(\alpha L - \gamma K)}$ >1

Relation between elasticities for a Production functions

$q = 5L + 3K$ $\qquad \frac{\partial q}{\partial K} = 3$

Output elasticity of K, L

Scale w.r.t. $\bar{q}$. $\qquad \frac{\partial q}{\partial L} = 5$

$e_K = \frac{K}{q}\frac{\partial q}{\partial K} = \frac{3K}{q}$

$e_L = \frac{5L}{q}$

Let $K = \bar{K}$, $L = \bar{L}$ $\quad q = \bar{q}$

$L \to \lambda\bar{L}$, $K \to \lambda\bar{K}$

$q = 5\lambda\bar{L} + 3\lambda\bar{K}$

$= \lambda(5\bar{L} + 3\bar{K})$

$\boxed{\frac{dq}{d\lambda} = \bar{q}}$

$\therefore \frac{dq}{d\lambda} = \bar{q}$

$e_\lambda = \frac{\lambda}{q}\frac{dq}{d\lambda} = \frac{\lambda}{\lambda \bar{q}}\bar{q} = 1$

… $= 1 = e_\lambda$

e_λ θ αr /

$$e_K + e_L = \frac{3K + 5L}{8} = \frac{8}{8} = 1 = e_\lambda$$

Sum of elasticities of K, L = Scale elasticity