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Number System

- Real
  - Rational
    - Integers
      - −ve
      - 0
      - +ve (Natural Nos.)
      - (0 and +ve: Whole Nos.)
    - Fractions / Decimals
  - Irrational
- Imaginary
- Complex (Real + Imaginary)

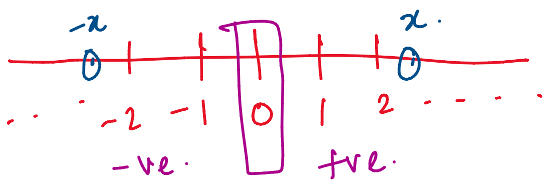
Fractions

- Proper
- Improper
- Mixed.

Decimals

- Terminating
- Recurring.
- Non-terminating & Non recurring (Irrational)

$\boxed{\text{REAL NOS}}$ — any no on the no line.



$(-x)^2 > 0$

$(-2)^2 = 4 > 0$

$x^2 > 0$

$2^2 = 4 > 0.$

Imp $\boxed{R^2 \geqslant 0}$ → Mathematical definition of a real No

There are 2 real nos a and b.

$a^2 + b^2 = 0$ Find a and b.

$\geqslant 0 + \geqslant 0$

$\therefore a^2 = 0$ and $b^2 = 0$. $\Rightarrow$ $a = b = 0$

$(x-2)^2 + (y-3)^2 + (z+4)^2 = 0$ find xyz.

$(x-2)^2 = 0 \Rightarrow x - 2 = 0$

$x = 2.$

$y = 3$

$x = 2$

$y = 3$

$xyz = 2 \times 3 \times (-4)$

$= -24.$

$x = 2$ $\quad$ $xyz = 2\times3\times(-4)$ $\quad$ $x=2$

$y = 3$ $\quad$ $= -24$. $\quad$ $y=3$

$z = -4$ $\quad$ $z=-4$

If $a^3+b^3+c^3 = 3abc$ and $a+b+c \neq 0$ prove that $a=b=c$.

$a^3+b^3 = \boxed{(a+b)(a^2-ab+b^2)}$

$= \boxed{(a+b)^3 - 3ab(a+b)}$

$(a^3+b^3)+c^3 - 3abc = 0$.

$(a+b)^3 - 3ab(a+b) + c^3 - 3abc = 0$

$[(a+b)+c][(a+b)^2-(a+b)c+c^2] - 3ab(a+b) - 3abc = 0$.

$x^3+y^3 = (x+y)(x^2-xy+y^2)$

$a+b = x$

$c = y$.

$(a+b+c)(a^2+2ab+b^2-ac-bc+c^2) - 3ab(a+b+c) = 0$

$(a+b+c)(a^2+b^2+c^2+2ab-ac-bc-3ab) = 0$.

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$.

$\neq 0$ $\quad$ $=0$

$(a^2+b^2+c^2-ab-bc-ca = 0)\times 2$.

$2a^2+2b^2+2c^2-2ab-2bc-2ca = 0$

$(a^2-2ab+b^2)+(b^2-2bc+c^2)+(a^2-2ac+c^2) = 0$

$(a-b)^2+(b-c)^2+(c-a)^2 = 0$

$=0$ $\quad$ $=0$ $\quad$ $=0$

$a-b=0$ $\quad$ $b-c=0$ $\quad$ $c-a=0$

$a=b$ $\quad$ $b=c$ $\quad$ $c=a$ $\quad$ $\therefore a=b=c$

Imp If the sum of 2 or more squares $=0$ then each square $=0$

V V Imp

$\boxed{\text{Arithmetic Mean} \geq \text{Geometric Mean}}$

x, y. $\quad$ Arithmetic Mean$(x,y) = \frac{x+y}{2}$.

Geometric Mean$(x,y) = \sqrt{xy}$.

a and b are 2 real nos

$\therefore (a-b)$ is also a real no

$\therefore (a-b)^2 \geq 0$ $\quad$ $[\because R^2 \geq 0]$.

$a^2-2ab+b^2 \geq 0$

$$a^2+b^2 \geq 2ab.$$

$$ab = \sqrt{a^2b^2}$$

$$\frac{a^2+b^2}{2} \geq ab.$$

$$\frac{a^2+b^2}{2} \geq \sqrt{a^2b^2}$$

V V Imp.

$$\boxed{AM(a^2, b^2) \geq GM(a^2, b^2)}$$

$a^2 \geq 0 \quad b^2 \geq 0.$

for any 2 Non-negative real nos.

$$\rightarrow \frac{a^2+b^2}{2} \geq ab.$$

$$\frac{a^2+b^2}{2} \geq (\underline{\quad})$$

$$\text{Min}\left(\frac{a^2+b^2}{2}\right) = ab.$$

$$ab \leq (\underline{\quad})$$

$$\frac{a^2+b^2}{2} \geq ab \leftarrow$$

$$\frac{a^2+b^2}{2} = \text{Max}(ab)$$

$$\text{Min}\left(\frac{a^2+b^2}{2}\right) = \text{Max}(ab) \Rightarrow \underline{\text{when } a=b}.$$

$$\frac{a^2+b^2}{2} = ab.$$

$$a^2+b^2 = 2ab.$$

$$a^2+b^2-2ab = 0$$

$$(a-b)^2 = 0$$

$$\boxed{a=b}$$

$\underline{a+b=10}$ find Maximum value of ab where a, b are Non negative integers.

a	$b = 10-a$	ab
0	10	0
1	9	9.
2	8	16.
3	7	21
4	6	24
5	5	25
6	4	24

If the sum of 2 Non-negative real nos is constant then their product will be maximum when the nos are equal.

find the minimum value of $(a+b)$

6 4 24

$ab = 36.$

find the minimum value of $(a+b)$ where a, b are +ve integers

a	$b = \frac{36}{a}$	$a+b$
1 +	36	37
2	18	20
3	12	15
4	9	13
6	6	12

If the product of 2 non-negative real numbers is constant then their sum will be minimum when the numbers are equal.

AM $\geq$ GM is applicable for more than 2 numbers

$$\frac{a_1+a_2+a_3+\dots+a_n}{n} \geq \sqrt[n]{a_1 a_2 a_3 \dots a_n}.$$

$a+b+c = 24$ find the maximum value of abc.

$a=b=c = \frac{24}{3} = 8.$

$\therefore abc = 8^3 = 512$

$\boxed{a+b+c+d = 30}$ where a, b, c, d are Natural Nos. (+ve integers)

find the minimum value of $(a-b)^2 + (a-c)^2 + (a-d)^2 = ?$ 2

$\frac{30}{4} = 7.5$

8, 8, 7, 7

Hint : the minimum value of the square of any no = 0

ideally $a-b=0$ $a-c=0$ $a-d=0$

$a=b$ $a=c$ $a=d.$

$a=b=c=d$

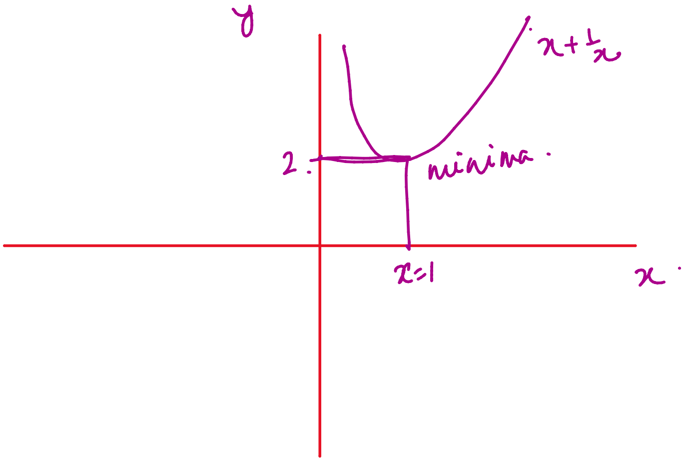
If x is a real non-negative number find the minimum value of $\left(x+\frac{1}{x}\right)$

Hint: apply AM $\geq$ GM.

$$\frac{x+\frac{1}{x}}{2} \geq \sqrt{x \times \frac{1}{x}}.$$

$x+\frac{1}{x} \geq 2$

$\text{Min}\left(x+\frac{1}{x}\right) = 2.$



Number System $\rightarrow$ AM $\geq$ GM