(i) To evaluate : $\underset{(x,y)\to(a,b)}{Lt} f(x,y)$ .... [Simultaneous Limit]

(ii) Repeated Limits:-

Consider $u = f(x,y)$ defined in the neighbourhood of (a,b)

$$\left.\begin{array}{l}\underset{x\to a}{Lt}\left\{\underset{y\to b}{Lt} f(x,y)\right\} = \lambda \\ \underset{y\to b}{Lt}\left\{\underset{x\to a}{Lt} f(x,y)\right\} = \lambda'\end{array}\right\}$$

Two Repeated Limit need not be the same.

Note: (i) If the simultaneous limit exists, then the two repeated limits (if they exist) are equal. Converse is not true.

(ii) If the repeated limits are not equal, then the simultaneous limit may not exist.

Q. $f(x,y) = \dfrac{xy}{\sqrt{x^2+y^2}}$. Find simultaneous and repeated limit at $(0,0)$.

Repeated Limit:

$$\underset{x\to 0}{Lt}\ \underset{y\to 0}{Lt} \frac{xy}{\sqrt{x^2+y^2}} = 0$$

$$\underset{y\to 0}{Lt}\ \underset{x\to 0}{Lt} \frac{xy}{\sqrt{x^2+y^2}} = 0$$

Simultaneous limit:

$$\underset{(x,y)\to(0,0)}{Lt} \frac{xy}{\sqrt{x^2+y^2}}$$

(r,θ)

$x = r\cos\theta$, $y = r\sin\theta$.

$x\to 0$, $y\to 0$, $r\to 0$,

$Lt \quad r^2 \sin\theta\cdot\cos\theta$

$x \to 0,\ y \to 0,\ r \to 0,$

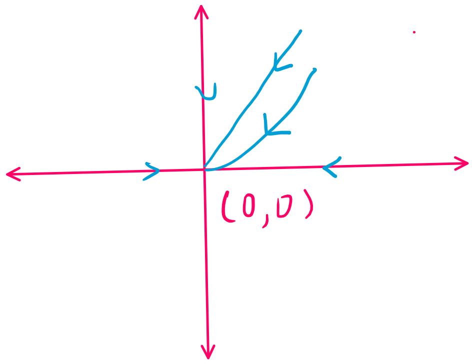
$$\lim_{r \to 0} \frac{r^2 \sin\theta \cdot \cos\theta}{r} = \lim_{r \to 0} r \sin\theta \cdot \cos\theta = 0.$$

Q. $\displaystyle \lim_{(x,y)\to(0,0)} xy \frac{x^2 - y^2}{x^2+y^2} = ?$ (a) 0 (b) 1 (c) -1 (d) None.

Fraction

$$x \cdot y \left(\frac{x^2 - y^2}{x^2+y^2} \right) \leq 1 \to 0.$$

$$\left| \frac{x^2 - y^2}{x^2+y^2} \right| \leq 1.$$



$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} =$$

$$\overset{\to 0}{x} \cdot \overset{\to 0}{y} \left(\frac{1}{\sqrt{x^2+y^2}} \right) \longrightarrow 0.$$

Definition of Limit:-

Suppose a fn $f(x,y)$ has limit l at pt (a,b).

Then: $|f(x,y) - l| < \varepsilon$ for any $\varepsilon > 0$

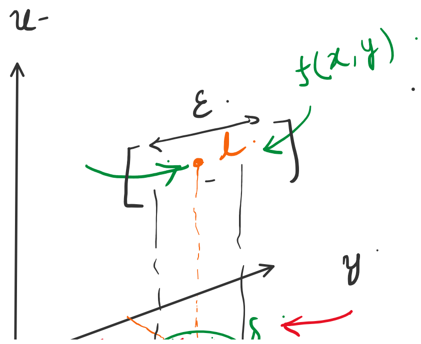
whenever: $(x-a)^2 + (y-b)^2 < \delta^2$.

i.e $|x-a| < \delta,\ |y-b| < \delta$.

$(x-a)^2 + (y-b)^2 < \delta^2$

[Circle around (a,b) with rad $= \delta$].

$|x-a| < \delta \quad |y-b| < \delta$



$$|x-a| < \delta, \quad |y-b| < \delta$$