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Numericals on Binomial, Poison and Normal Distribution

Q1. What is the probability of getting exactly 2 even numbers, when a balanced die is rolled 3 times?

$n = 3$ times (no. of trials)

x = number of even no.s

p = success of getting even no. $= \frac{3}{6} = 1/2$

$q = 1-p = 1/2$

p.m.f of Binomial distribution is

$$P(X=x) = f(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$P(x=2) = f(2) = {}^{3}C_{2} \left(1/2\right)^{2} \left(1/2\right)^{3-2}$$

$$= {}^{3}C_{2}\ \frac{1}{2^2} \cdot \frac{1}{2}$$

$$= \frac{3!}{2!\,1!}\ \frac{1}{8}$$

$$= \frac{3 \times \cancel{2!}}{\cancel{2!}} \times \frac{1}{8} = \frac{3}{8} \text{ (ans)}$$

Q2 If a random variable x follows binomial distribution with mean $=2$ and $E(x^2) = \frac{28}{5}$, find $P(x \neq 0)$

$E(x) = np = 2$ — ①

$P(x \neq 0) = 1 - P[x=0]$

$= 1 - {}^{n}C_{0}\, p^{0} q^{n-0}$

$E(x) = np = 2$ — (1) $\qquad P(X \neq 0) = 1 - \ldots$

$E(x^2) = \frac{28}{5}$ $\qquad = 1 - {}^{n}C_0\, p^0 q^{n-0}$

$V(x) = E(x^2) - E(x)^2$

$= \frac{28}{5} - 2^2$

$V(x) = \frac{28 - 20}{5} = \frac{8}{5}$

$V(x) = npq = \frac{8}{5}$ — (2)

$\Rightarrow 2q = 8/5$

$q = \frac{\cancel{8}\,4}{5 \times \cancel{2}} = 4/5$

$\therefore p = 1 - q = \frac{1}{5}$

$np = 2$

$n = \frac{2}{p} = 2 \times 5 = 10$

$n = 10, \quad p = 1/5, \quad q = 4/5$

$\therefore P(x \neq 0) = 1 - {}^{10}C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10-0}$

$= 1 - 1 \times 1 \times \left(\frac{4}{5}\right)^{10}$

$= 1 - \left(\frac{4}{5}\right)^{10}$ (ans)

Q3. A person takes a step forward with probability 0.25 and backward with probability 0.75. What is the probability that at the end of 7 steps he will be one step away from the will be one step away from ... starting point?

$n = 7$ $\quad p = 0.25 \quad q = 0.75$

$x = 3$ or 4

$P(X = 3 \text{ or } 4) = P(X=3) + P(X=4)$

$= {}^7C_3 (0.25)^3 (0.75)^4 + {}^7C_4 (0.25)^4 (0.75)^3$

$= (\quad\quad)$ ans.

(5) If a random variable X follows Poisson Distribution satisfying $2 \cdot P(X=0) = P(X=1)$, determine $P(X>0)$, $P(X>0 \mid X<2)$ and $E(X)$. Given $e^{-2} = 0.1353$.

given $2P(X=0) = P(X=1)$

p.m.f $f(x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

$2 \cdot \cancel{e^{-\lambda}} \dfrac{\lambda^0}{0!} = \cancel{e^{-\lambda}} \dfrac{\lambda^1}{1!}$

$\boxed{2 = \lambda}$

$\therefore P(X>0) = 1 - P(X=0) = 1 - \dfrac{e^{-\lambda}\lambda^0}{0!} = 1 - \dfrac{e^{-2} \cdot 1}{0!}$

$= 1 - e^{-2}$

$= 1 - 0.1353 = 0.8647$

$= 1 - 0.1353 = 0.8647$ (ans)

(ii) $P(X>0 \mid X<2) = \dfrac{P(X>0 \cap X<2)}{P(X<2)}$

$= \dfrac{P(X=1)}{\{P(0)+P(X=1)\}}$

$= \dfrac{e^{-2}.2}{\{e^{-2}+2e^{-2}\}} = \dfrac{2\cancel{e^{-2}}}{3\cancel{e^{-2}}}$

$= 2/3$ (ans)

(iii) $E(X) = \lambda = 2$ (ans).

(6) The manufacturer of a certain electronic component knows that 3% of his product is defective. He sells the components in boxes of 100 and guarantees that not more than 3% in any box will be defective. What is the probability that a box will fail to meet the guarantee? (Given $e^3 = 20.1$)

$$\boxed{\left.\begin{array}{l} n \to \infty \\ p \to 0 \end{array}\right\} np \to \lambda \qquad B.D \to P.D}$$

$p = 0.03$

n quite large $= 100$

$np(=\lambda) = 0.03 \times 100 = 3$

$$P(X=x) = f(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-3}\,3^x}{x!}$$

$$P(X=x)=f(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-3}3^x}{x!}$$

$$P(X>3) = 1 - P(X \le 3) = 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$$

$$= 1 - e^{-3}\frac{3^0}{0!} - e^{-3}\frac{3^1}{1!} - e^{-3}\frac{3^2}{2!} - e^{-3}\frac{3^3}{3!}$$

$$1 - e^{-3} - e^{-3}\cdot 3 - e^{-3}\frac{9}{2} - e^{-3}\frac{\cancel{27}\,9}{\cancel{6}\,2}$$

$$1 - e^{-3}\left[1 + 3 + \frac{9}{2} + \frac{9}{2}\right]$$

$$1 - 13e^{-3} = 1 - \frac{13}{e^3} = 1 - \frac{13}{20.1}$$

$$= 1 - 0.647$$

$$= 0.353$$

cars)

(7) A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day follows Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used.
(ii) some demand is refused.

$\lambda = 1.5$

(1) $P(x=0) = e^{-1.5}\frac{1.5^0}{0!} = e^{-1.5}$

(2) $P(X>2) = 1 - P(X \le 2) = 1 - [P(x=0) + P(x=1)]$

⋮

Q8 The probability of recovering from a certain disease is 0·75. Write down the probability distribution of no. of recoveries among 4 such patients. Find mean and s.d.

$${}^nC_x p^x q^{n-x}$$

$$x = 0 \cdots n$$

$n = 4$

$p = 0·75$ $\quad x = 0, 1, 2, 3, 4$

p.m.f is given by

$$P(X = x) = f(x) = {}^4C_x (0·75)^x (0·25)^{4-x} \quad \text{for } x = 0,1,2,3,4$$

$$= 0 \quad \text{elsewhere.}$$

$$\text{Mean} = E(X) = n \cdot p = 4 \times 0·75 = 3·00$$

$$V(X) = npq = 3 \times 0·25 = 0·75$$

Q9. The probability that an individual will suffer a bad reaction from a particular injection is 0·001. Determine the probability that out of 2000 individuals more than 2 individuals will suffer a bad reaction.

n is large size $= 2000$ $\therefore \lambda = np = 2000 \times 0.001$

$p \to 0$ ie $p = 0.001$ $= 2$

$$P(x>2) = 1 - P(x \le 2)$$
$$= 1 - [f(0) + f(1) + f(2)]$$
$$= 1 - \left[e^{-2} \left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} \right] \right]$$
$$= 1 - e^{-2} (1 + 2 + 2)$$
$$= 1 - 5e^{-2} \text{ Ans)}$$