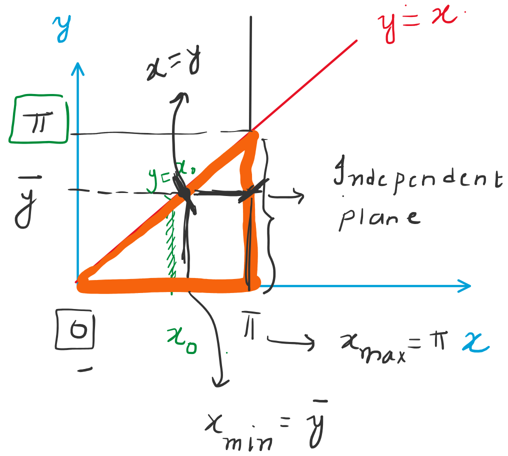
Q. $$\int_0^{\pi}\int_0^{x} \frac{\sin y}{(\pi - y)}\, dy\, dx = ?$$

$y \in [0, x]$, $x \in [0, \pi]$.



$$\int_0^{\pi}\int_0^{x} f(x,y)\, dy\, dx = \int_0^{\pi}\int_y^{\pi} \quad dx\, dy$$

$$\int_0^{\pi}\int_y^{\pi} \frac{\sin y}{(\pi - y)}\, dx\, dy = \int_0^{\pi} \left\{ \int_y^{\pi} \frac{\sin y}{(\pi - y)}\, dx \right\} dy$$

$$= \int_0^{\pi} \frac{\sin y}{\cancel{(\pi - y)}} \cancel{(\pi - y)} \, dy$$

$$= \int_0^{\pi} \sin y \, dy = -\left[\cos y\right]_0^{\pi}$$

$$= -\left[-1 - 1\right] = 2 .$$

Q. Evaluate: $\iint_R |xy|\, dx\, dy$, where $R = \{(x,y) \text{ s.t } x^2 + 4y^2 \geq 1 \ \& \ x^2 + y^2 \leq 1\}$

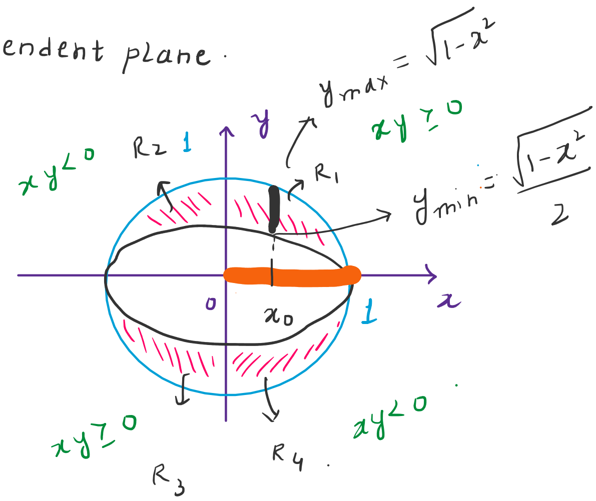
Independent plane.

Circle: $x^2 + y^2 = 1$.

Ellipse: $x^2 + 4y^2 = 1$.

$\hookrightarrow$ $C(0,0)$

x-int : 1, y-int : $1/2$.



$$f(x,y) = |xy| = \begin{cases} xy, & xy \geq 0 \\ -xy, & xy < 0 \end{cases}$$

$$\iint_R |xy|\, dx\, dy = \iint_{R_1} xy + \iint_{R_2} -xy + \iint_{R_3} xy + \iint_{R_4} -xy$$

$1 \quad \sqrt{1-x^2}$

$= 4 \iint$

R $\quad$ R1 $\quad$ R2 $\quad$ R3 $\quad$ R4

$$= 4 \iint_{R_1} xy \, dx\, dy = 4 \int_0^1 \int_{\frac{\sqrt{1-x^2}}{2}}^{\sqrt{1-x^2}} (xy) \, dy \, dx .$$

$$= 4 \int_0^1 x \cdot \left[\frac{y^2}{2} \right]_{\frac{\sqrt{1-x^2}}{2}}^{\sqrt{1-x^2}} dx$$

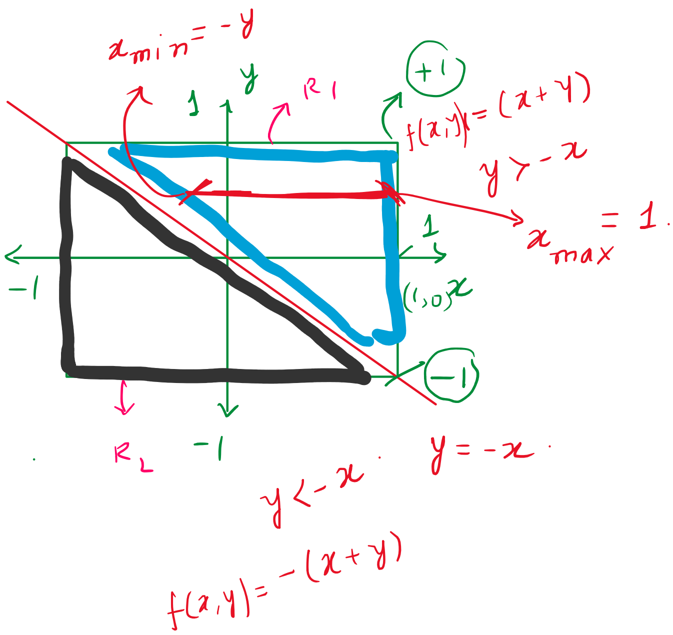
$$= 2 \int_0^1 x \left[(1-x^2) - \frac{(1-x^2)}{4} \right] dx$$

$$= 2 \int_0^1 x \left[\frac{3}{4} (1-x^2) \right] dx = \frac{3}{2} \int_0^1 x - x^3 \, dx$$

$$= \frac{3}{2} \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{3}{2} \left[\frac{1}{2} - \frac{1}{4} \right] = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8} .$$

Q. Evaluate: $\int_{-1}^{1} \int_{-1}^{1} |x+y| \, dx \, dy$

$$|x+y| = \begin{cases} (x+y), & x+y \geq 0 \Rightarrow y \geq -x \\ -(x+y), & x+y < 0 \Rightarrow y < -x \end{cases}$$



$$\int_{-1}^{1} \int_{-1}^{1} |xy| \, dx \, dy = \iint_{R_1} (x+y) + \iint_{R_2} -(x+y) .$$

$$= 2 \iint_{R_1} (x+y) \, dx \, dy .$$

$$= 2 \int_{-1}^{1} \int_{-y}^{1} (x+y) \, dx \, dy \qquad [HW!] .$$

HW

Q. $\int_0^1 \int_x^1 y^4 e^{xy^2} \, dy \, dx = ?$