Functions Questions

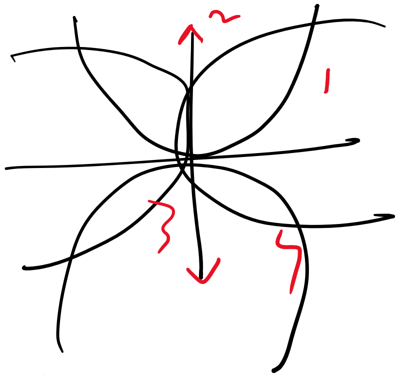
Thursday, November 2, 2023 5:36 PM

9062395123

Functions..

$f(x) = x^2 = y$

$\bigcirc{f(x)}$ $\quad y^2 = x$



Butterfly function

Functional formation + Graphical representations done alike..

Power of a function $\quad f^{20}$ -

$y_n = f(x) = x^{2n}$

$y_1 = f(x) = x^2$

$y_2 = f(x) = x^4$

$y_3 = f(x) = x^9$

$y = \log_{\textcircled{a}} \textcircled{x}$ $\xrightarrow{} f(a,x)$

$\boxed{0 < x < 1}$

$x > 1$

$\left[\left[\left[\left[[x]\right]\right]\right]\right]$

$y = f(f(f(f(x))))$

2.99

0.99

$y = \textcircled{2}.78 \leftarrow$ fractional part

G.I.F Greatest Integer Function

2.99
2.09
2.50

$y = \textcircled{2}.\underline{78}$ ← Fraction Part

GIF Greatest Integer Function

Integral Part

$[[x]] \to x$ $\quad [2.78] \to 2$

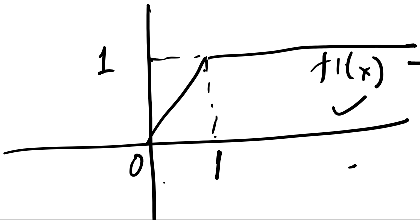
$x = 2.78$ $\quad [x] \to 2$

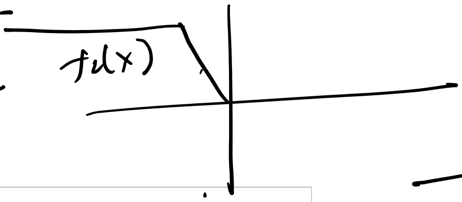
$[[x]] \to x$ $\quad [[[x]]] = [x]$

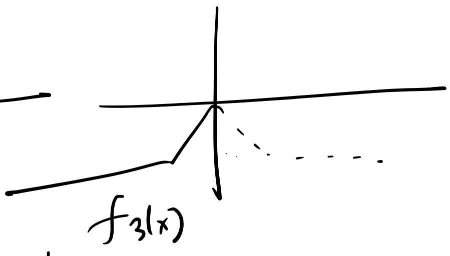
$[x]$

$2 \neq 2.78$

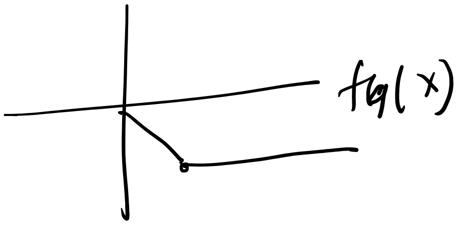
#







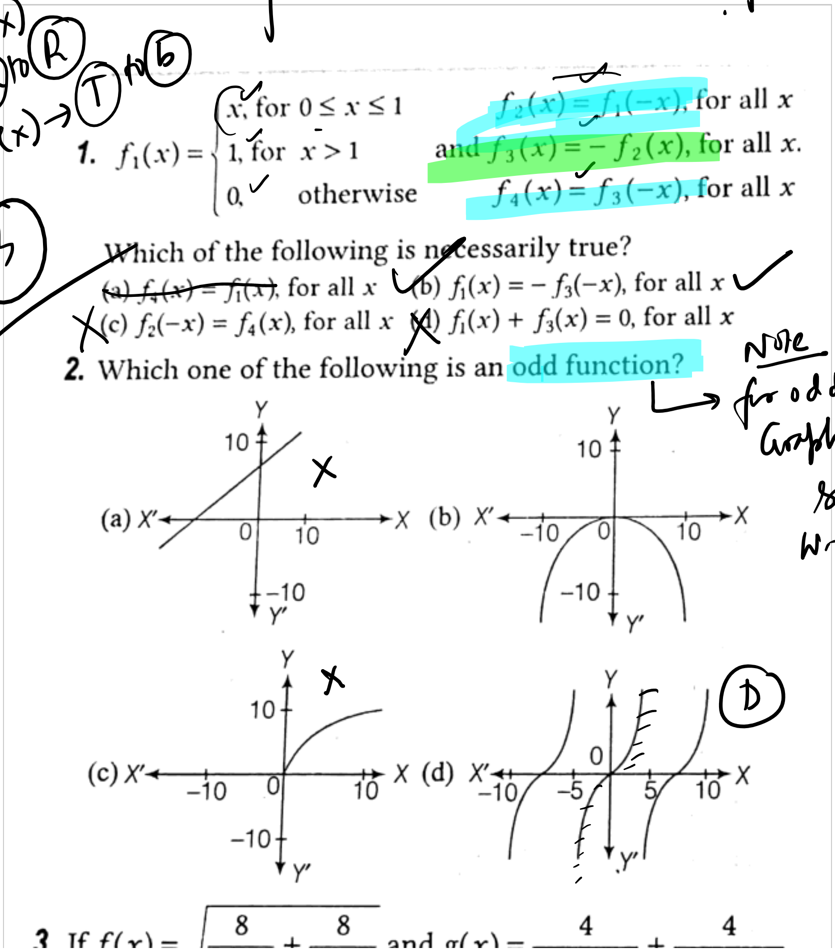
Functions Questions



f(-x) D to R

-f(x) → T to B

B



1. $f_1(x) = \begin{cases} x, \text{ for } 0 \le x \le 1 \\ 1, \text{ for } x > 1 \\ 0, \text{ otherwise} \end{cases}$ $\quad f_2(x) = f_1(-x)$, for all x and $f_3(x) = -f_2(x)$, for all x. $f_4(x) = f_3(-x)$, for all x

Which of the following is necessarily true?

(a) ~~$f_4(x) = f_1(x)$~~, for all x (b) $f_1(x) = -f_3(-x)$, for all x

(c) $f_2(-x) = f_4(x)$, for all x (d) $f_1(x) + f_3(x) = 0$, for all x

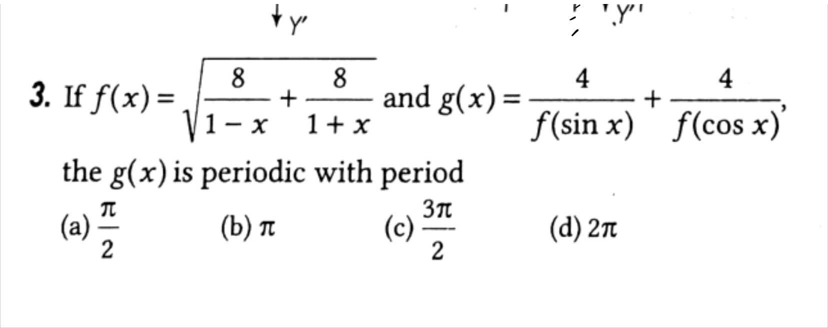
2. Which one of the following is an odd function?

3. If $f(x) = \sqrt{\frac{8}{\quad} + \frac{8}{\quad}}$ and $g(x) = \frac{4}{\quad} + \frac{4}{\quad}$

Note for odd function Graph will be Symmetrical w.r.t. ORIGIN

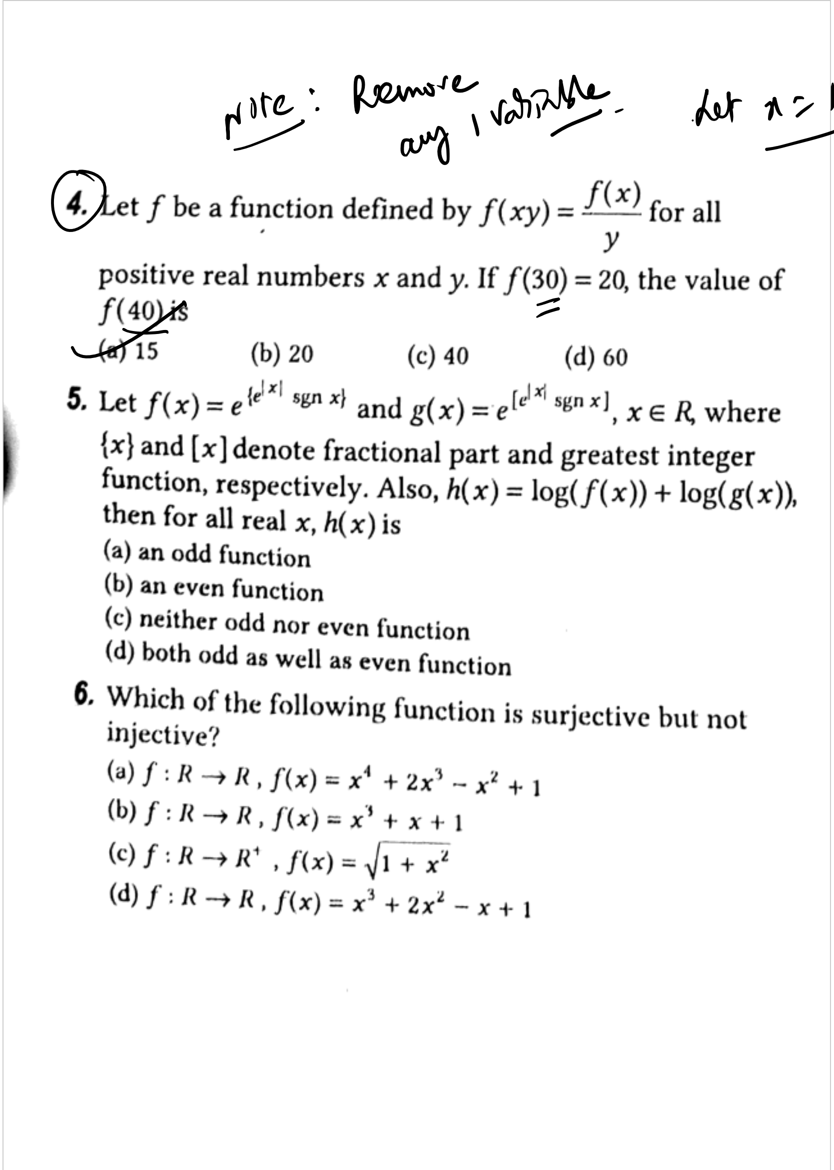
$f(x) = f(-x)$ Even

$f(x) = -f(-x)$ odd



3. If $f(x)=\sqrt{\dfrac{8}{1-x}+\dfrac{8}{1+x}}$ and $g(x)=\dfrac{4}{f(\sin x)}+\dfrac{4}{f(\cos x)}$, the $g(x)$ is periodic with period

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π



note: Remove any 1 variable.

Let $x=1$

$f(y)=\dfrac{f(1)}{y}$

$f(30)=\dfrac{f(1)}{30}=20$ $f(1)=600$

$f(40)=\dfrac{f(1)}{40}=\dfrac{600}{40}=\underline{15}$

4. Let f be a function defined by $f(xy)=\dfrac{f(x)}{y}$ for all positive real numbers x and y. If $f(30)=20$, the value of $f(40)$ is

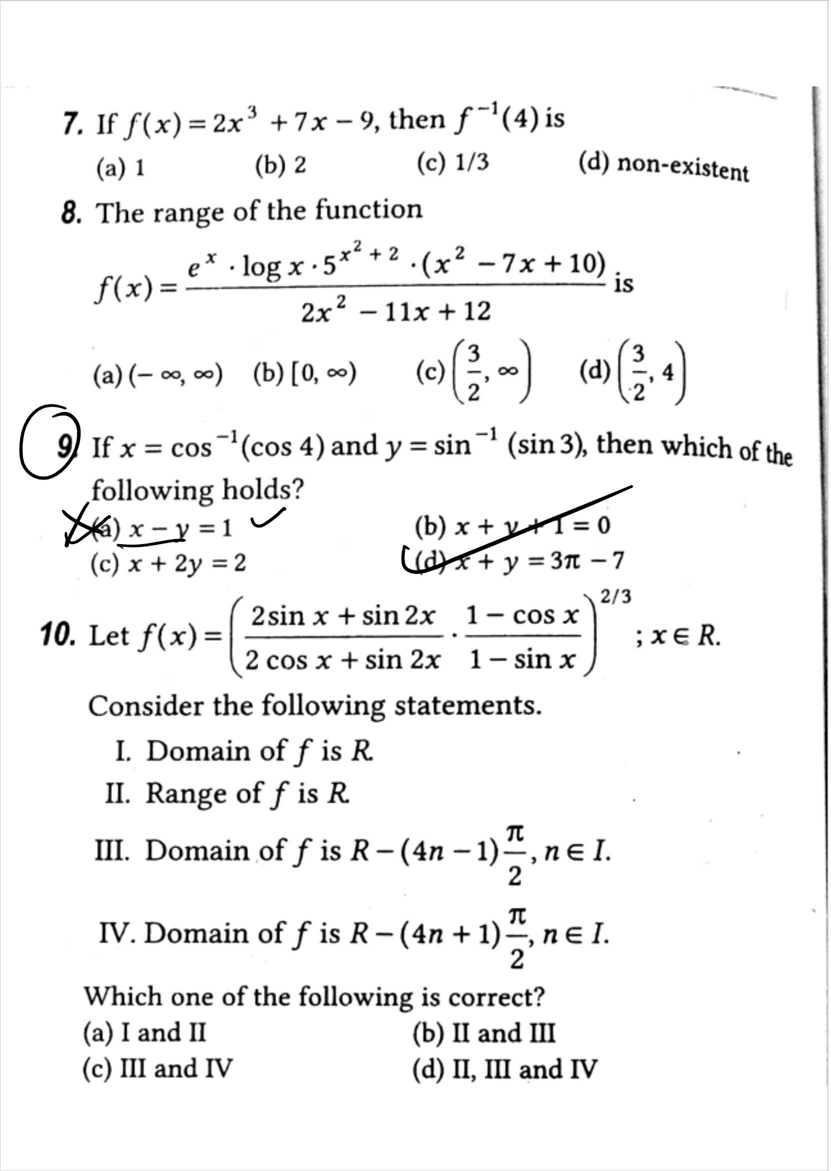
[✓ (a) 15] (b) 20 (c) 40 (d) 60

5. Let $f(x)=e^{\{e^{|x|}\operatorname{sgn} x\}}$ and $g(x)=e^{[e^{|x|}\operatorname{sgn} x]}$, $x\in R$, where $\{x\}$ and $[x]$ denote fractional part and greatest integer function, respectively. Also, $h(x)=\log(f(x))+\log(g(x))$, then for all real x, $h(x)$ is

(a) an odd function
(b) an even function
(c) neither odd nor even function
(d) both odd as well as even function

6. Which of the following function is surjective but not injective?

(a) $f:R\to R$, $f(x)=x^4+2x^3-x^2+1$
(b) $f:R\to R$, $f(x)=x^3+x+1$
(c) $f:R\to R^+$, $f(x)=\sqrt{1+x^2}$
(d) $f:R\to R$, $f(x)=x^3+2x^2-x+1$



7. If $f(x) = 2x^3 + 7x - 9$, then $f^{-1}(4)$ is

(a) 1 (b) 2 (c) 1/3 (d) non-existent

8. The range of the function

$$f(x) = \frac{e^x \cdot \log x \cdot 5^{x^2+2} \cdot (x^2 - 7x + 10)}{2x^2 - 11x + 12} \text{ is}$$

(a) $(-\infty, \infty)$ (b) $[0, \infty)$ (c) $\left(\frac{3}{2}, \infty\right)$ (d) $\left(\frac{3}{2}, 4\right)$

9. If $x = \cos^{-1}(\cos 4)$ and $y = \sin^{-1}(\sin 3)$, then which of the following holds?

✗ (a) $x - y = 1$ ✓ (b) $x + y + 1 = 0$

(c) $x + 2y = 2$ (d) $x + y = 3\pi - 7$

10. Let $f(x) = \left(\frac{2\sin x + \sin 2x}{2\cos x + \sin 2x} \cdot \frac{1 - \cos x}{1 - \sin x}\right)^{2/3}$; $x \in R$.

Consider the following statements.

I. Domain of f is R.

II. Range of f is R.

III. Domain of f is $R - (4n-1)\frac{\pi}{2}, n \in I$.

IV. Domain of f is $R - (4n+1)\frac{\pi}{2}, n \in I$.

Which one of the following is correct?

(a) I and II (b) II and III

(c) III and IV (d) II, III and IV

$x = 4$ ✗

$y = 3$

$x = \cos^{-1}(\cos(2\pi - 4)) = 2\pi - 4$

$y = \sin^{-1}(\sin(\pi - 3))$

$= \pi - 3$

11. If $f(x) = e^{\sin(x-[x])\cos \pi x}$, where $[x]$ denotes the greatest integer function, then $f(x)$ is
(a) non-periodic
(b) periodic with no fundamental period
(c) periodic with period 2
(d) periodic with period π

12. The range of the function, $f(x) = \cot^{-1}(\log_{0.5}(x^4 - 2x^2 + 3))$ is
(a) $(0, \pi)$ (b) $\left(0, \frac{3\pi}{4}\right]$ (c) $\left[\frac{3\pi}{4}, \pi\right)$ (d) $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$

13. Range of $f(x) = \left[\frac{1}{\log(x^2+e)}\right] + \frac{1}{\sqrt{1+x^2}}$, where $[\cdot]$ denotes greatest integer function, is
(a) $\left(0, \frac{e+1}{e}\right) \cup \{2\}$ (b) $(0, 1)$
(c) $(0, 1] \cup \{2\}$ (d) $[0, 1) \cup \{2\}$

Number   [illegible] Part

14. The period of the function $f(x) = \sin(x + 3 - [x+3])$, where $[\cdot]$ denotes greatest integer function, is
(a) $2\pi + 3$ (b) 2π (c) 1 (d) 4

$f(x+1) = f(x)$

N

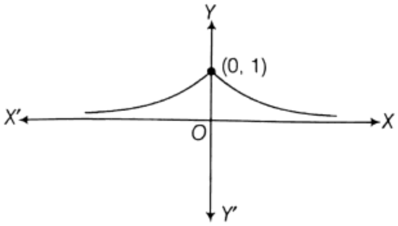
Let, $x + 3 = t$

$f(t) = \sin\{\ \}$
$= \sin\{t - [t]\}$
$= \sin\{t\}$

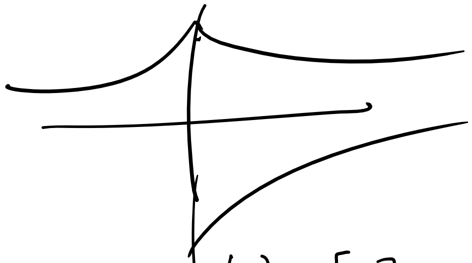
Note

of a function comes down to fractional part period is always → ①

15. Which one of the following functions best represent the graph as shown below?



$y = e^{-|x|}$



(a) $f(x) = \frac{1}{1+x^2}$ (b) $f(x) = \frac{1}{\sqrt{1+|x|}}$
(c) $f(x) = e^{-|x|}$ (d) $f(x) = a^{|x|}, a > 1$

$\{x\} + [x] = x$

$\{x\} = \frac{1}{[x]}$

$x - [x] = \frac{1}{[x]}$

$x = [x] + \frac{1}{[x]}$

$\boxed{x > 2}$

$1 + \frac{1}{1} = 2$

$2 + \frac{1}{2} > 2$

$(N) - \{1\}$

16. The solution set for $[x]\{x\} = 1$, where $\{x\}$ and $[x]$ denote fractional part and greatest integer functions, is
(a) $R^+ - (0, 1)$ (b) $R^+ - \{1\}$
(c) $\left\{m + \frac{1}{m}, m \in I - \{0\}\right\}$ (d) $\left\{m + \frac{1}{m}, m \in N - \{1\}\right\}$

(a) $R^+ - (0, 1)$ (b) $R^+ - \{1\}$

(c) $\left\{m + \frac{1}{m}, m \in I - \{0\}\right\}$ (d) $\left\{m + \frac{1}{m}, m \in N - \{1\}\right\}$

$x = [x] + \frac{1}{[x]}$

$x = m + \frac{1}{m}$ $(N) - \{1\}$

17. The domain of definition of function

$f(x) = \log(\sqrt{x^2 - 5x - 24} - x - 2)$, is

(a) $(-\infty, -3]$ (b) $(-\infty - 3] \cup [8, \infty)$

(c) $\left(-\infty, \frac{-28}{9}\right)$ (d) None of these

$\ln -5x \ln 0$

$\ln x$ $(x>0)$

So, here $\sqrt{x^2-5x-24} > x+2$

Case I

$x^2-5x-24 \geq 0$ $x+2<0$

$(x-8)(x+3) \geq 0$ $x<-2$

$x \leq -3$

Case II

$x^2-5x-24 \geq 0$, $x+2 \geq 0$

$x^2-5x-24 > (x+2)^2$

$\cancel{x^2}-5x-24 > \cancel{x^2}+4x+4$

$9x < -28$ $x < -28/9$

$x \in \emptyset$

No common sol^n

$x \in (-\infty, -3]$

18. If $f(x)$ is a function $f : R \rightarrow R$, we say $f(x)$ has property

I. If $f(f(x)) = x$ for all real numbers x.

II. If $f(-f(x)) = -x$ for all real numbers x.

How many linear functions, have both property I and II?

(a) 0 (b) 2

(c) 3 (d) Infinite

19. Let $f(x) = \frac{x}{1+x}$ and $g(x) = \frac{rx}{1-x}$. Let S be the set of all real numbers r, such that $f(g(x)) = g(f(x))$ for infinitely many real numbers x. The number of elements in set S is

(a) 1 (b) 2

(c) 3 (d) 5

20. Let $f(x)$ be linear functions with the properties that $f(1) \leq f(2)$, $f(3) \geq f(4)$ and $f(5) = 5$. Which one of the following statements is true?

(a) $f(0) < 0$ (b) $f(0) = 0$ ✗

(c) $f(1) < f(0) < f(-1)$ (d) $f(0) = 5$ ✓

$y = mx + b$

$m \geq 0$

$f(3) \geq f(4)$

$m \leq 0$

$m = 0$

$f(5) = 5$

$b = 5$

now $y = b$

$f(x) = b$

$f(5) = b = 5$

Here we have to know the function is Constant

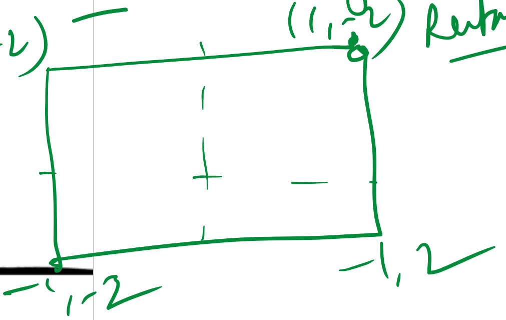
21. Suppose R is relation whose graph is symmetric to both X-axis and Y-axis and that the point $(1, 2)$ is on the graph of R. Which one of the following is not necessarily on the graph of R?

(a) $(-1, 2)$ (b) $(1, -2)$

(c) $(-1, -2)$ (d) $(2, 1)$

(Visualisation) Both

$(1,2)$ $(1,-2)$



Rectangle

$y = [x] + \{x\}$

0

$0 \leq y < 1$ $\{y\} = y$

$0 \leq [x] + 1 < 2$

$-1 \leq [x] < 1$

when $-1 \leq x < 0 \Rightarrow y = 0$

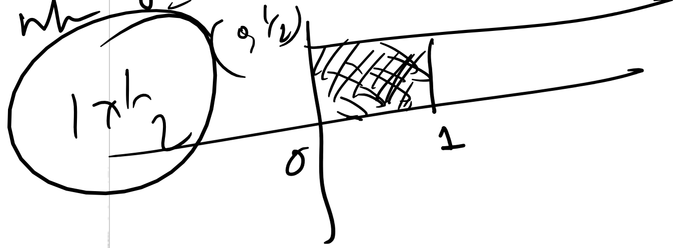
when $0 \leq x < 1 \Rightarrow y = \frac{1}{2}$

$(0, \frac{1}{2})$ $y = \frac{1}{2}$

22. The area between the curve $2\{y\} = [x] + 1, 0 \leq y < 1$, where $\{\cdot\}$ and $[\cdot]$ are the fractional part and greatest when 0 ≤ x < 1 0 < y = x/2

22. The area between the curve $2\{y\} = [x] + 1, 0 \le y < 1$, where $\{\cdot\}$ and $[\cdot]$ are the fractional part and greatest integer functions, respectively and the X-axis, is

[✓ (a) $\frac{1}{2}$] (b) 1 (c) 0 (d) $\frac{3}{2}$



23. If $f(x) = \sin^{-1} x$ and $g(x) = [\sin(\cos x)] + [\cos(\sin x)]$, then range of $f(g(x))$ is (where, $[\cdot]$ denotes greatest integer function)

(a) $\left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$ (b) $\left\{\frac{-\pi}{2}, 0\right\}$

(c) $\left\{0, \frac{\pi}{2}\right\}$ (d) $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

24. The number of solutions of the equation $e^{2x} + e^x - 2 = [\{x^2 + 10x + 11\}]$ is (where, $\{x\}$ denotes fractional part of x and $[x]$ denotes greatest integer function)

(a) 0 (b) 1 (c) 2 (d) 3

25. Total number of values of x, of the form $\frac{1}{n}, n \in N$ in the interval $x \in \left[\frac{1}{25}, \frac{1}{10}\right]$, which satisfy the equation $\{x\} + \{2x\} + \ldots + \{12x\} = 78x$ (where, $\{\cdot\}$ represents fractional part function), is

(a) 12 [✓ (b) 13]
(c) 14 (d) 15

$6 \quad \frac{12(13)}{2} \quad 78$

~~13 n 25~~ 13

$\{x\} = x - [x]$

$\{x\} + \{2x\} + \{3x\} + \dots + [12x] = 78x$

$x - [x] + 2x - [2x] + 3x - [3x] + \dots + 12x - [12x] = 78x$

$78x - \{[x] + [2x] + \dots + [12x]\} = \cancel{78x}$

$[x] + [2x] + \dots + [12x]$

$0 \le 12x < 1$

$0 \le x < \frac{1}{12}$ $\forall \frac{1}{25} < x \le \frac{1}{10}$

Common value $\in \left[\frac{1}{25}, \frac{1}{12}\right)$

26. The sum of the maximum and minimum values of the function $f(x) = \dfrac{1}{1 + (2\cos x - 4\sin x)^2}$ is

(a) $\frac{22}{21}$ (b) $\frac{21}{20}$ (c) $\frac{22}{20}$ (d) $\frac{21}{11}$

27. Let $f: X \to Y$, $f(x) = \sin x + \cos x + 2\sqrt{2}$ be invertible, then $X \to Y$ is/are

(a) $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to [\sqrt{2}, 3\sqrt{2}]$

(b) $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right] \to [\sqrt{2}, 3\sqrt{2}]$

(c) $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right] \to [\sqrt{2}, -3\sqrt{2}]$

(d) $\left[-\frac{3\pi}{4}, -\frac{\pi}{4}\right] \to [\sqrt{2}, 3\sqrt{2}]$

28. The range of values of a, so that all the roots of the equation $2x^3 - 3x^2 - 12x + a = 0$ are real and distinct, belongs to

(a) (7, 20) (b) (−7, 20)
(c) (−20, 7) (d) (−7, 7)

belongs to

(a) (7, 20) (b) (−7, 20)

(c) (−20, 7) (d) (−7, 7)

29. If $f(x)$ is continuous such that $|f(x)| \leq 1, \forall\ x \in R$ and $g(x) = \dfrac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}$, then range of $g(x)$ is

(a) $[0, 1]$ (b) $\left[0, \dfrac{e^2+1}{e^2-1}\right]$

(c) $\left[0, \dfrac{e^2-1}{e^2+1}\right]$ (d) $\left[\dfrac{1-e^2}{1+e^2}, 0\right]$

30. Let $f(x) = \sqrt{|x| - \{x\}}$, where {} denotes the fractional part of x and X, Y and its domain and range respectively, then

(a) $f : X \rightarrow Y : y = f(x)$ is one-one function

(b) $X \in \left(-\infty, -\dfrac{1}{2}\right] \cup [0, \infty)$ and $Y \in \left[\dfrac{1}{2}, \infty\right)$

(c) $X \in \left(-\infty, -\dfrac{1}{2}\right] \cup [0, \infty)$ and $Y \in [0, \infty)$

(d) None of the above

31. If the graphs of the functions $y = \ln x$ and $y = ax$ intersect at exactly two points, then a must be

(a) $(0, e)$ (b) $\left(\dfrac{1}{e}, 0\right)$

(c) $\left(0, \dfrac{1}{e}\right)$ (d) None of these

32. A quadratic polynomial maps from $[-2, 3]$ onto $[0, 3]$ and touches X-axis at $x = 3$, then the polynomial is

(a) $\dfrac{3}{16}(x^2 - 6x + 16)$ (b) $\dfrac{3}{25}(x^2 - 6x + 9)$

(c) $\dfrac{3}{25}(x^2 - 6x + 16)$ (d) $\dfrac{3}{16}(x^2 - 6x + 9)$

33. The range of the function $y = \sqrt{2\{x\} - \{x\}^2 - \frac{3}{4}}$ (where, $\{\cdot\}$ denotes the fractional part) is

(a) $\left[-\frac{1}{4}, \frac{1}{4}\right]$
(b) $\left[0, \frac{1}{2}\right)$
(c) $\left[0, \frac{1}{4}\right]$
(d) $\left[\frac{1}{4}, \frac{1}{2}\right]$

34. Let $f(x)$ be a fourth differentiable function such that $f(2x^2 - 1) = 2xf(x), \forall\ x \in R$, then $f^{iv}(0)$ is equal to (where, $f^{iv}(0)$ represents fourth derivative of $f(x)$ at $x = 0$)

(a) 0
(b) 1
(c) -1
(d) Data insufficient

35. Number of solutions of the equation $[y + [y]] = 2\cos x$ is (where, $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ and $[\cdot]$ denotes the greatest integer function)

(a) 1
(b) 2
(c) 3
(d) None of these

36. If a function satisfies $f(x+1)+f(x-1)=\sqrt{2}f(x)$, then period of $f(x)$ can be

(a) 2 (b) 4 (c) 6 (d) 8

37. If x and α are real, then the inequation

$\log_2 x + \log_x 2 + 2\cos\alpha \le 0$

(a) has no solution

(b) has exactly two solutions

(c) is satisfied for any real α and any real x in (0, 1)

(d) is satisfied for any real α and any real x in $(1, \infty)$

38. The range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|} = x^2 - a$ is satisfied for maximum number of values of 'x'

(a) $(-\infty, -1)$ (b) $(-\infty, \infty)$ (c) $(-1, 1)$ (d) $(-1, \infty)$

39. Let $f : R \to R$ be a function defined by $f(x) = \{|\cos x|\}$, where $\{x\}$ represents fractional part of x. Let S be the set containing all real values x lying in the interval $[0, 2\pi]$ for which $f(x) \neq |\cos x|$. The number of elements in the set S is

(a) 0 (b) 1 (c) 3 (d) infinite

40. The domain of the function

$f(x) = \sqrt{\log_{\sin x + \cos x}(|\cos x| + \cos x)}, 0 \le x \le \pi$ is

(a) $(0, \pi)$ (b) $\left(0, \frac{\pi}{2}\right)$

(c) $\left(0, \frac{\pi}{3}\right)$ (d) None of these

41. If $f(x) = (x^2 + 2\alpha x + \alpha^2 - 1)^{1/4}$ has its domain and range such that their union is set of real numbers, then α satisfies

(a) $-1 < \alpha < 1$
(b) $\alpha \leq -1$
(c) $\alpha \geq 1$
(d) $\alpha \leq 1$

42. Let $f : (e, \infty) \to R$ be a function defined by $f(x) = \log(\log(\log x))$, the base of the logarithm being e. Then,

(a) f is one-one and onto
(b) f is one-one but not onto
(c) f is onto but not one-one
(d) the range of f is equal to its domain

43. The expression $x^2 - 4px + q^2 > 0$ for all real x and also $r^2 + p^2 < qr$, the range of $f(x) = \dfrac{x+r}{x^2 + qx + p^2}$ is

(a) $\left[\dfrac{p}{2r}, \dfrac{q}{2r}\right]$
(b) $(0, \infty)$
(c) $(-\infty, 0)$
(d) $(-\infty, \infty)$

44. Let $f(x) = \dfrac{x^4 - \lambda x^3 - 3x^2 + 3\lambda x}{x - \lambda}$. If range of $f(x)$ is the set of entire real numbers, the true set in which λ lies is

(a) $[-2,2]$
(b) $[0,4]$
(c) $(1,3)$
(d) None of these

45. Let $a = 3^{1/224} + 1$ and for all $n \geq 3$,

let $f(n) = {}^{n}C_0\, a^{n-1} - {}^{n}C_1 a^{n-2} + {}^{n}C_2 a^{n-3}$

$$+ \ldots + (-1)^{n-1} \cdot {}^{n}C_{n-1} \cdot a^0.$$

If the value of $f(2016) + f(2017) = 3^K$, the value of K is

(a) 6 (b) 8
(c) 9 (d) 10

46. The area bounded by $f(x) = \sin^{-1}(\sin x)$ and

$$g(x) = \frac{\pi}{2} - \sqrt{\frac{\pi^2}{2} - \left(x - \frac{\pi}{2}\right)^2}$$ is

(a) $\frac{\pi^3}{8}$ sq units (b) $\frac{\pi^2}{8}$ sq units

(c) $\frac{\pi^3}{2}$ sq units (d) $\frac{\pi^2}{2}$ sq units

47. If $f : R \rightarrow R$, $f(x) = \frac{x^2 + bx + 1}{x^2 + 2x + b}$, $(b > 1)$ and $f(x)$, $\frac{1}{f(x)}$ have the same bounded set as their range, the value of b is

(a) $2\sqrt{3} - 2$ (b) $2\sqrt{3} + 2$
(c) $2\sqrt{2} - 2$ (d) $2\sqrt{2} + 2$

48. The period of $\sin \frac{\pi [x]}{12} + \cos \frac{\pi [x]}{4} + \tan \frac{\pi [x]}{3}$, where $[x]$ represents the greatest integer less than or equal to x is

(a) 12 (b) 4
(c) 3 (d) 24

49. If $f(2x+3y, 2x-7y) = 20x$, then $f(x, y)$ equals

(a) $7x-3y$ (b) $7x+3y$

(c) $3x-7y$ (d) $x-y$

50. The range of the function $f(x) = \sqrt{x-1} + 2\sqrt{3-x}$ is

(a) $[\sqrt{2}, 2\sqrt{2}]$ (b) $[\sqrt{2}, \sqrt{10}]$

(c) $[2\sqrt{2}, \sqrt{10}]$ (d) $[1, 3]$

51. The domain of the function

$$f(x) = \cos^{-1}(\sec(\cos^{-1} x)) + \sin^{-1}(\operatorname{cosec}(\sin^{-1} x)) \text{ is}$$

(a) $x \in R$ (b) $x = 1, -1$

(c) $-1 \le x \le 1$ (d) $x \in \phi$

52. Let $f(x)$ be a polynomial one-one function such that $f(x)f(y) + 2 = f(x) + f(y) + f(xy), \forall\ x, y \in R - \{0\}$, $f(1) \ne 1, f'(1) = 3$.

Let $g(x) = \dfrac{x}{4}(f(x)+3) - \int_0^x f(x)\, dx$, then

(a) $g(x) = 0$ has exactly one root for $x \in (0, 1)$

(b) $g(x) = 0$ has exactly two roots for $x \in (0, 1)$

(c) $g(x) \ne 0,\ \forall\ x \in R - \{0\}$

(d) $g(x) = 0,\ \forall\ x \in R - \{0\}$

53. Let $f(x)$ be a polynomial with real coefficients such that $f(x) = f'(x) \times f'''(x)$. If $f(x) = 0$ is satisfied $x = 1, 2, 3$ only, then the value of $f'(1)\, f'(2)\, f'(3)$ is

(a) positive (b) negative

(c) 0 (d) Inadequate data

54. Let $A = \{1, 2, 3, 4, 5\}$ and $f : A \to A$ be an into function such that $f(i) \neq i, \forall\ i \in A$, then number of such functions f are

(a) 1024 (b) 904

(c) 980 (d) None of these

55. If functions $f : \{1, 2, \dots, n\} \to \{1995, 1996\}$ satisfying $f(1) + f(2) + \dots + f(1996) =$ odd integer are formed, the number of such functions can be

(a) 2^n (b) $2^{n/2}$ (c) n^2 (d) 2^{n-1}

56. The range of $y = \sin^3 x - 6\sin^2 x + 11\sin x - 6$ is

(a) $[-24, 2]$ (b) $[-24, 0]$

(c) $[0, 24]$ (d) None of these

57. Let $f(x) = x^2 - 2x$ and $g(x) = f(f(x) - 1) + f(5 - f(x))$, then

(a) $g(x) < 0, \forall x \in R$

(b) $g(x) < 0$, for some $x \in R$

(c) $g(x) \geq 0$, for some $x \in R$

(d) $g(x) \geq 0, \forall x \in R$

58. If $f(x)$ and $g(x)$ are non-periodic functions, then $h(x) = f(g(x))$ is

(a) non-periodic
(b) periodic
(c) may be periodic
(d) always periodic, if domain of $h(x)$ is a proper subset of real numbers

59. If $f(x)$ is a real-valued function discontinuous at all integral points lying in $[0, n]$ and if $(f(x))^2 = 1$, $\forall\ x \in [0, n]$, then number of functions $f(x)$ are

(a) 2^{n+1} (b) 6×3^n (c) $2 \times 3^{n-1}$ (d) 3^{n+1}

60. A function f from integers to integers is defined as

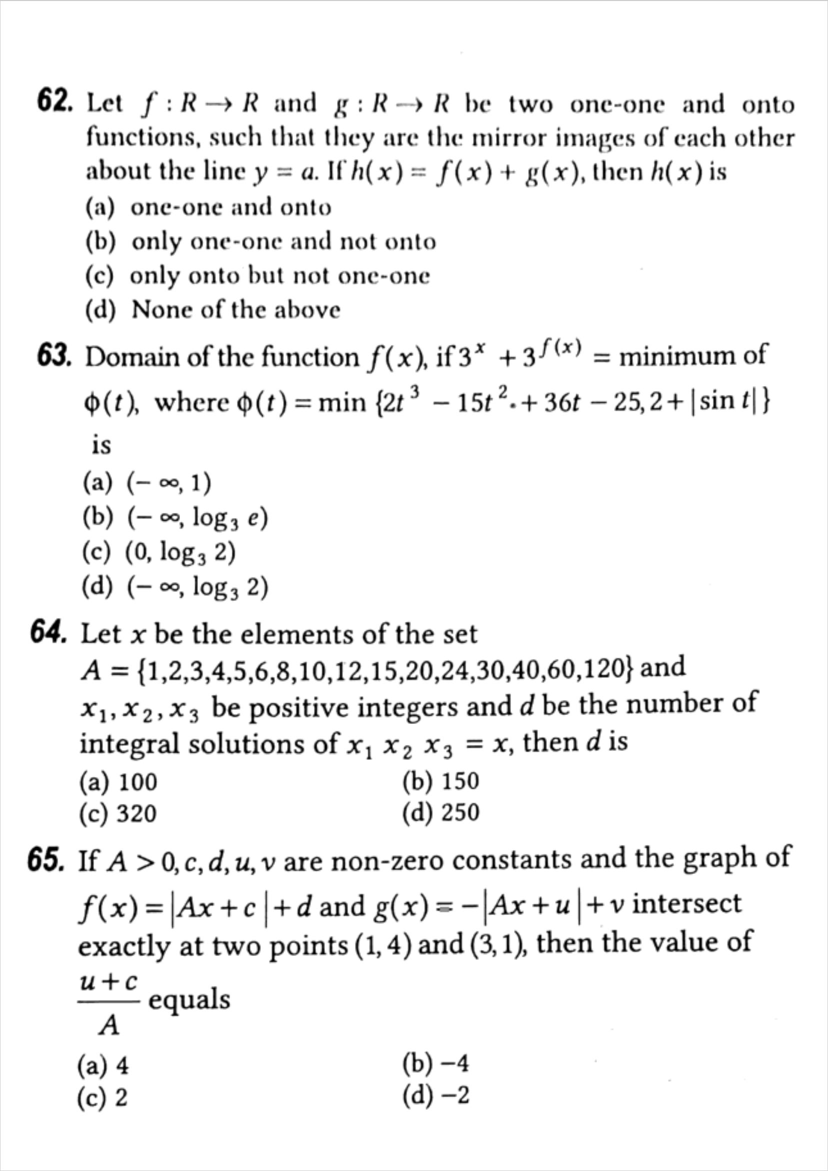
$$f(x) = \begin{cases} n+3, & n \in \text{odd} \\ n/2, & n \in \text{even} \end{cases}$$

Suppose $k \in$ odd and $f(f(f(k))) = 27$, then the sum of digits of k is

(a) 3 (b) 6 (c) 9 (d) 12

61. If $f : R \to R$ and $f(x) = \dfrac{\sin(\pi\{x\})}{x^4 + 3x^2 + 7}$, where { } is a fractional part of x, then

(a) f is injective
(b) f is not one-one and non-constant
(c) f is a surjective
(d) f is a zero function



62. Let $f : R \rightarrow R$ and $g : R \rightarrow R$ be two one-one and onto functions, such that they are the mirror images of each other about the line $y = a$. If $h(x) = f(x) + g(x)$, then $h(x)$ is

(a) one-one and onto
(b) only one-one and not onto
(c) only onto but not one-one
(d) None of the above

63. Domain of the function $f(x)$, if $3^x + 3^{f(x)} =$ minimum of $\phi(t)$, where $\phi(t) = \min\{2t^3 - 15t^2 + 36t - 25, 2 + |\sin t|\}$ is

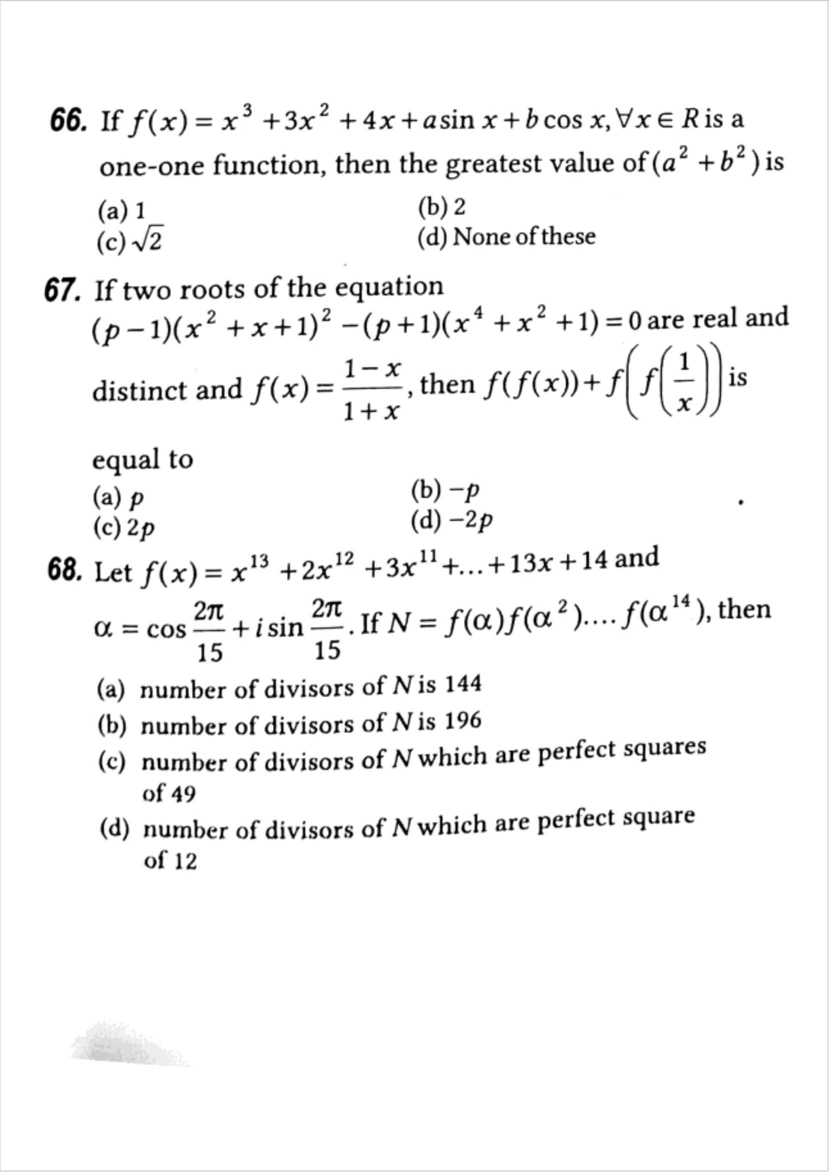
(a) $(-\infty, 1)$
(b) $(-\infty, \log_3 e)$
(c) $(0, \log_3 2)$
(d) $(-\infty, \log_3 2)$

64. Let x be the elements of the set $A = \{1,2,3,4,5,6,8,10,12,15,20,24,30,40,60,120\}$ and x_1, x_2, x_3 be positive integers and d be the number of integral solutions of $x_1 x_2 x_3 = x$, then d is

(a) 100 (b) 150
(c) 320 (d) 250

65. If $A > 0, c, d, u, v$ are non-zero constants and the graph of $f(x) = |Ax + c| + d$ and $g(x) = -|Ax + u| + v$ intersect exactly at two points $(1, 4)$ and $(3, 1)$, then the value of $\frac{u+c}{A}$ equals

(a) 4 (b) -4
(c) 2 (d) -2



66. If $f(x) = x^3 + 3x^2 + 4x + a\sin x + b\cos x, \forall x \in R$ is a one-one function, then the greatest value of $(a^2 + b^2)$ is

(a) 1
(b) 2
(c) $\sqrt{2}$
(d) None of these

67. If two roots of the equation $(p-1)(x^2 + x + 1)^2 - (p+1)(x^4 + x^2 + 1) = 0$ are real and distinct and $f(x) = \frac{1-x}{1+x}$, then $f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right)$ is equal to

(a) p
(b) $-p$
(c) $2p$
(d) $-2p$

68. Let $f(x) = x^{13} + 2x^{12} + 3x^{11} + \ldots + 13x + 14$ and $\alpha = \cos\frac{2\pi}{15} + i\sin\frac{2\pi}{15}$. If $N = f(\alpha)f(\alpha^2) \ldots . f(\alpha^{14})$, then

(a) number of divisors of N is 144
(b) number of divisors of N is 196
(c) number of divisors of N which are perfect squares of 49
(d) number of divisors of N which are perfect square of 12

69. The sum of the maximum and minimum values of function $f(x) = \sin^{-1} 2x + \cos^{-1} 2x + \sec^{-1} 2x$ is

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) $\frac{3\pi}{2}$

70. The complete set of values of 'a' for which the function $f(x) = \tan^{-1}(x^2 - 18x + a) > 0, \forall\ x \in R$, is

(a) $(81, \infty)$ (b) $[81, \infty)$

(c) $(-\infty, 81)$ (d) $(-\infty, 81]$

71. The domain of the function

$$f(x) = \sin^{-1} \frac{1}{|x^2 - 1|} + \frac{1}{\sqrt{\sin^2 x + \sin x + 1}}$$ is

(a) $(-\infty, \infty)$

(b) $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

(c) $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty) \cup \{0\}$

(d) None of the above

72. The domain of $f(x) = \dfrac{\log(\sin^{-1}\sqrt{x^2 + x + 1})}{\log(x^2 - x + 1)}$ is

(a) $(-1, 1)$ (b) $(-1, 0) \cup (0, 1)$

(c) $(-1, 0) \cup \{1\}$ (d) None of these

73. The domain of $f(x) = \sqrt{\sin^{-1}(3x - 4x^3)} + \sqrt{\cos^{-1} x}$ is equal to

(a) $\left[-1, -\frac{\sqrt{3}}{2}\right] \cup \left[0, \frac{\sqrt{3}}{2}\right]$ (b) $\left[-1, -\frac{1}{2}\right] \cup \left[0, \frac{1}{2}\right]$

(c) $\left[0, \frac{1}{2}\right]$ (d) None of these

74. The domain of the function

$f(x) = \sqrt[6]{4^x + 8^{2/3(x-2)} - 52 - 2^{2(x-1)}}$ is

(a) (0, 1) (b) [3, ∞)

(c) [1, 0) (d) None of these

75. The domain of derivative of the function $f(x) = |\sin^{-1}(2x^2 - 1)|$ is

(a) (−1, 1) (b) $(-1, 1) \sim \left\{0, \pm \frac{1}{\sqrt{2}}\right\}$

(c) (−1, 1) ~ {0} (d) $(-1, 1) \sim \left\{\pm \frac{1}{\sqrt{2}}\right\}$

76. The range of a function

$f(x) = \tan^{-1} \{\log_{5/4}(5x^2 - 8x + 4)\}$ is

(a) $\left(\frac{-\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left[\frac{-\pi}{4}, \frac{\pi}{2}\right)$

(c) $\left(\frac{-\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[\frac{-\pi}{4}, \frac{\pi}{2}\right]$

77. Which of the following function(s) is/are transcendal?

(a) $f(x) = 5\sin(\sqrt{x})$ (b) $f(x) = \dfrac{2\sin 3x}{x^2 + 2x - 1}$

(c) $f(x) = \sqrt{x^2 + 2x + 1}$ (d) $f(x) = (x^2 + 3) \cdot 2^x$

78. Let $f(x) = \dfrac{\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1} \cdot x$, then

(a) domain of $f(x)$ is $x \geq 1$ (b) domain of $f(x)$ is $[1, \infty) - \{2\}$

(c) $f'(10) = 1$ (d) $f'\left(\dfrac{3}{2}\right) = -1$

79. $f(x) = \cos^2 x + \cos^2\left(\dfrac{\pi}{3} + x\right) - \cos x \cdot \cos\left(x + \dfrac{\pi}{3}\right)$ is

(a) an odd function (b) an even function

(c) a periodic function (d) $f(0) = f(1)$

80. If the following functions are defined from $[-1, 1]$ to $[-1, 1]$, identify these which are into.

(a) $\sin(\sin^{-1} x)$ (b) $\dfrac{2}{\pi} \cdot \sin^{-1}(\sin x)$

(c) $\operatorname{sgn}(x) \cdot \log(e^x)$ (d) $x^3 \operatorname{sgn}(x)$

81. Let $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ x - 4, & x \geq 3 \end{cases}$

and $g(x) = \begin{cases} x - 3, & x < 4 \\ x^2 + 2x + 2, & x \geq 4 \end{cases}$, which one of the following is/are true?

(a) $(f + g)(3.5) = 0$ (b) $f(g(3)) = 3$

(c) $f(g(2)) = 1$ (d) $(f - g)(4) = 0$

82. If $f(x) = x^2 - 2ax + a(a+1)$, $f:[a, \infty) \to [a, \infty)$. If one of the solutions of the equation $f(x) = f^{-1}(x)$ is 5049, the other may be

(a) 5051 (b) 5048 (c) 5052 (d) 5050

83. The function g defined by $g(x) = \sin\alpha + \cos\alpha - 1; \alpha = \sin^{-1}\sqrt{\{x\}}$, where {} denotes fractional part function, is

(a) an even function (b) periodic function

(c) odd function (d) neither even nor odd

84. The graph of $f: R \to R$ defined by $y = f(x)$ is symmetric with respect to $x = a$ and $x = b$. Which of the following is true?

(a) $f(2a - x) = f(x)$ (b) $f(2a + x) = f(-x)$

(c) $f(2b + x) = f(-x)$ (d) f is periodic

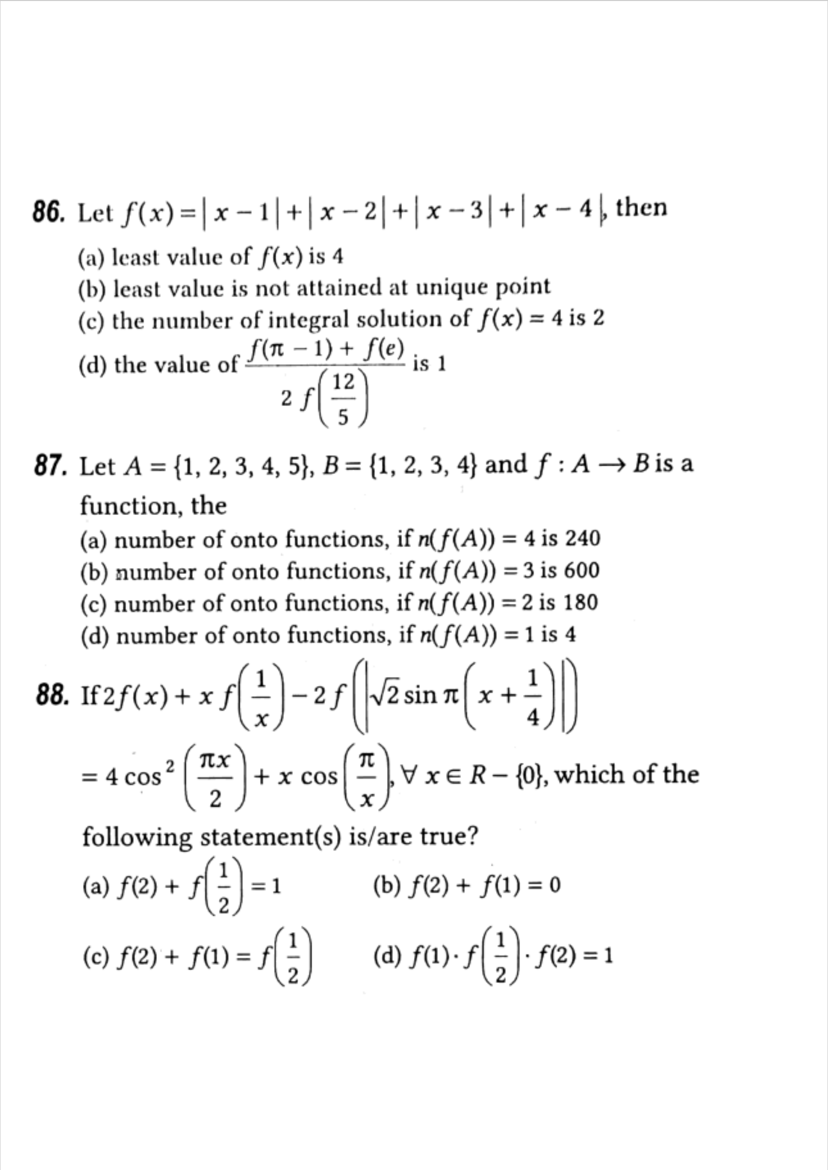
85. Let f be the continuous and differentiable function such that $f(x) = f(2 - x), \forall x \in R$ and $g(x) = f(1 + x)$, then

(a) $g(x)$ is an odd function

(b) $g(x)$ is an even function

(c) $f(x)$ is symmetric about $x = 1$

(d) None of the above



86. Let $f(x) = |x-1| + |x-2| + |x-3| + |x-4|$, then

(a) least value of $f(x)$ is 4

(b) least value is not attained at unique point

(c) the number of integral solution of $f(x) = 4$ is 2

(d) the value of $\dfrac{f(\pi-1)+f(e)}{2f\left(\dfrac{12}{5}\right)}$ is 1

87. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 4\}$ and $f : A \rightarrow B$ is a function, the

(a) number of onto functions, if $n(f(A)) = 4$ is 240

(b) number of onto functions, if $n(f(A)) = 3$ is 600

(c) number of onto functions, if $n(f(A)) = 2$ is 180

(d) number of onto functions, if $n(f(A)) = 1$ is 4

88. If $2f(x) + x\,f\left(\dfrac{1}{x}\right) - 2f\left(\left|\sqrt{2}\sin\pi\left(x+\dfrac{1}{4}\right)\right|\right)$

$= 4\cos^2\left(\dfrac{\pi x}{2}\right) + x\cos\left(\dfrac{\pi}{x}\right)$, $\forall\, x \in R - \{0\}$, which of the following statement(s) is/are true?

(a) $f(2) + f\left(\dfrac{1}{2}\right) = 1$

(b) $f(2) + f(1) = 0$

(c) $f(2) + f(1) = f\left(\dfrac{1}{2}\right)$

(d) $f(1) \cdot f\left(\dfrac{1}{2}\right) \cdot f(2) = 1$

89. If $f(x)$ is a differentiable function satisfying the condition $f(100x) = x + f(100x - 100), \forall\ x \in R$ and $f(100) = 1$, then $f(10^4)$ is

(a) 5049 (b) $\sum_{r=1}^{100} r$ (c) $\sum_{r=2}^{100} r$ (d) 5050

90. If $[x]$ denotes the greatest integer function then the extreme values of the function
$f(x) = [1 + \sin x] + [1 + \sin 2x] + \ldots + [1 + \sin nx], n \in I^+$,
$x \in (0, \pi)$ are

(a) $(n - 1)$ (b) n (c) $(n + 1)$ (d) $(n + 2)$

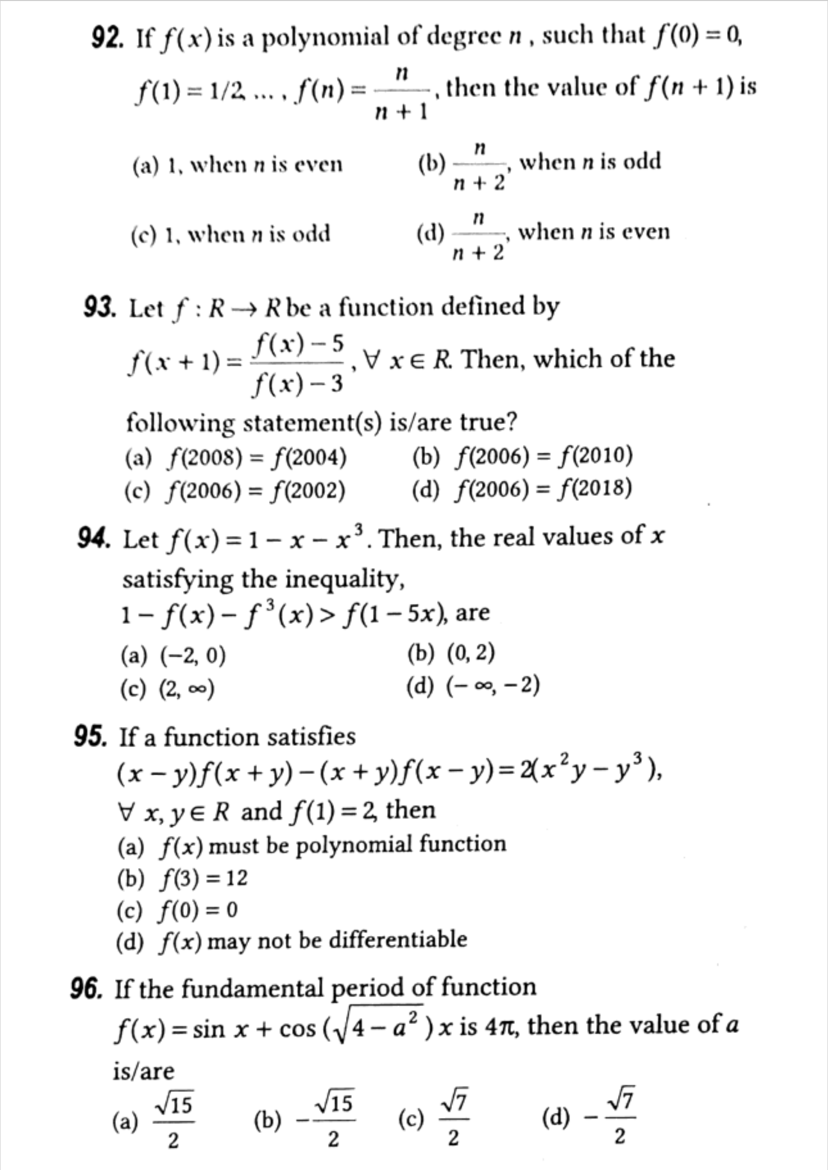
91. Which of the following is/are periodic?

(a) $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$

(b) $f(x) = \begin{cases} x - [x], & 2n \le x < 2n + 1 \\ \frac{1}{2}, & 2n + 1 \le x < 2n + 2 \end{cases}$, where $[\cdot]$ denotes the greatest integer function

(c) $f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$, where $[\cdot]$ denotes the greatest integer function

(d) $f(x) = ax - [ax + a] + \tan\left(\frac{\pi x}{2}\right)$, where $[\cdot]$ denotes the greatest integer function



92. If $f(x)$ is a polynomial of degree n , such that $f(0)=0$, $f(1)=1/2, \dots, f(n)=\dfrac{n}{n+1}$, then the value of $f(n+1)$ is

(a) 1, when n is even
(b) $\dfrac{n}{n+2}$, when n is odd
(c) 1, when n is odd
(d) $\dfrac{n}{n+2}$, when n is even

93. Let $f: R \to R$ be a function defined by $f(x+1)=\dfrac{f(x)-5}{f(x)-3}, \forall\ x \in R$. Then, which of the following statement(s) is/are true?

(a) $f(2008)=f(2004)$
(b) $f(2006)=f(2010)$
(c) $f(2006)=f(2002)$
(d) $f(2006)=f(2018)$

94. Let $f(x)=1-x-x^3$. Then, the real values of x satisfying the inequality,
$1-f(x)-f^3(x)>f(1-5x)$, are

(a) $(-2, 0)$
(b) $(0, 2)$
(c) $(2, \infty)$
(d) $(-\infty, -2)$

95. If a function satisfies
$(x-y)f(x+y)-(x+y)f(x-y)=2(x^2y-y^3)$,
$\forall\ x, y \in R$ and $f(1)=2$, then

(a) $f(x)$ must be polynomial function
(b) $f(3)=12$
(c) $f(0)=0$
(d) $f(x)$ may not be differentiable

96. If the fundamental period of function $f(x)=\sin x+\cos(\sqrt{4-a^2})\,x$ is 4π, then the value of a is/are

(a) $\dfrac{\sqrt{15}}{2}$
(b) $-\dfrac{\sqrt{15}}{2}$
(c) $\dfrac{\sqrt{7}}{2}$
(d) $-\dfrac{\sqrt{7}}{2}$

97. Let $f(x)$ be a real valued function such that $f(0) = \frac{1}{2}$ and $f(x+y) = f(x)f(a-y) + f(y)f(a-x)$, $\forall\ x, y \in R$, then for some real a,

(a) $f(x)$ is a periodic function

(b) $f(x)$ is a constant function

(c) $f(x) = \frac{1}{2}$ (d) $f(x) = \frac{\cos x}{2}$

98. If $f(g(x))$ is one-one function, then

(a) $g(x)$ must be one-one (b) $f(x)$ must be one-one

(c) $f(x)$ may not be one-one (d) $g(x)$ may not be one-one

99. Which of the following functions have their range equal to R (the set of real numbers)?

(a) $x \sin x$

(b) $\frac{[x]}{\tan 2x} \cdot x \in \left(-\frac{\pi}{4} \cdot \frac{\pi}{4}\right) - \{0\}$, where $[\cdot]$ denotes the greatest integer function

(c) $\frac{x}{\sin x}$

(d) $[x] + \sqrt{\{x\}}$, where $[\cdot]$ and $\{\cdot\}$, respectively denote the greatest integer and fractional part functions

100. Which of the following pairs of function are identical?

(a) $f(x) = e^{\ln \sec^{-1} x}$ and $g(x) = \sec^{-1} x$

(b) $f(x) = \tan(\tan^{-1} x)$ and $g(x) = \cot(\cot^{-1} x)$

(c) $f(x) = \text{sgn}(x)$ and $g(x) = \text{sgn}(\text{sgn}(x))$

(d) $f(x) = \cot^2 x \cdot \cos^2 x$ and $g(x) = \cot^2 x - \cos^2 x$

101. Let $f : R \to R$ defined by $f(x) = \cos^{-1}(-\{-x\})$, where $\{x\}$ denotes fractional part of x. Then, which of the following is/are correct?

(a) f is many one but not even function

(b) Range of f contains two prime numbers

(c) f is non-periodic

(d) Graph of f does not lie below X-axis