Ug 15th June only Questions

Thursday, June 15, 2023 5:39 PM

22 th June for 88 done

Theory of Equations

$ax^2+bx+c=0$

α, β

$\alpha+\beta$

Root

$(x-3)(x+2)=0$

$x+2=0$

$x-3=0$

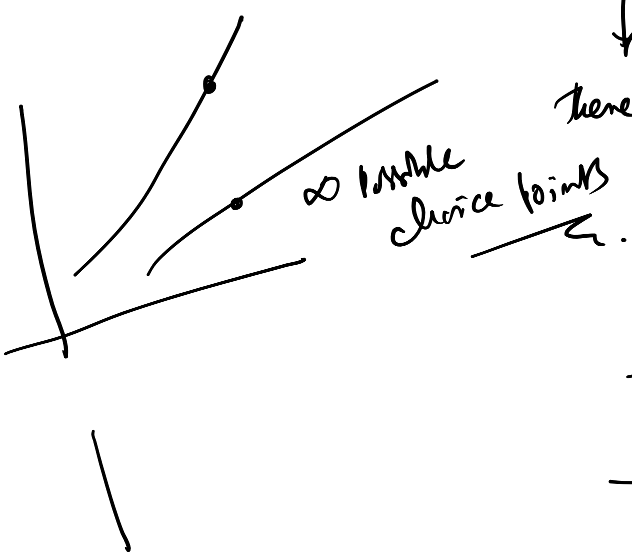
Whenever an intersection

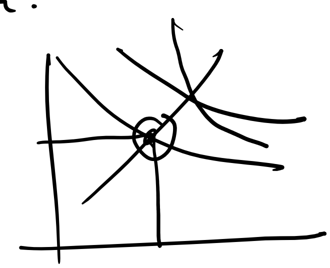
↓

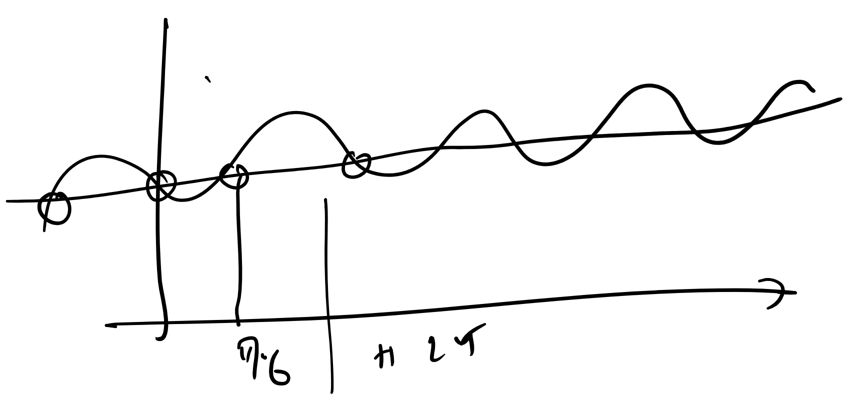
there is a soln

If you need soln

You need an Interse







$ax^2+bx+c=0$

[illegible] $=0$

$ax^2+bx+c=0$

$ax^3+bx^2+cx+d=0$

$(ax^{15}+bx^{14}+cx^{13}+dx^{12}+\dots+0_1) \Rightarrow 0$

Sum of the roots

$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\dots$

$\sum \alpha_i = \alpha_1+\alpha_2+\dots+\alpha_n = (-1)^1 \frac{a_1}{a_0}$

in case (4) $\sum \alpha_i\alpha_j = (\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4)$

$\sum \alpha_i\alpha_j = (\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\dots+\alpha_1\alpha_n)+(\alpha_2\alpha_3+\alpha_2\alpha_4+\dots+\alpha_2\alpha_n)+\dots+(\alpha_{n-1}\alpha_n)$

$= (-1)^2 \frac{a_2}{a_0}$

$\sum \alpha_1\alpha_2\alpha_3 = (-1)^3 \frac{a_3}{a_0}$

$\sum \alpha_1\alpha_2\alpha_3\alpha_4 = (-1)^4 \frac{a_4}{a_0}$

Given $2a+3b+6c=0$ $\boxed{a,b,c \in R}$

Show that $ax^2+bx+c=0$ has at least 1 root b/w 0 & 1

find antiderivative of this value

$2a+3b+6c=0$

$\left(\frac{a}{3}+\frac{b}{2}+c\right)=0$

$f'(x) = ax^2+bx+c$

$f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx+d$

$f(0)=d$

$f(1)=\left(\frac{a}{3}+\frac{b}{2}+c+d\right)$

$= 0+d = d$

$f(0)=d=f(1)$

Rolle's theorem

for answer is

∴ the root must be b/w (0) & (1)

Rolle ... the an... ∴ the root must be b/w 0 & 1

Ug 15th June only ...

$D = b^2 - 4ac = \underline{\underline{121}}\ (\underline{\underline{a+b+c}})^2 + \underline{\underline{24}}(a - \underline{\underline{b}})^2 > 0$

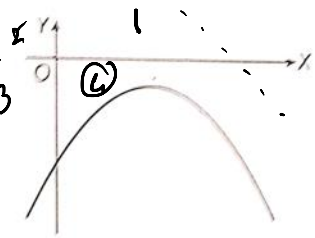
1. If a, b, c are real and $a \neq b$, the roots of the equation $2(a-b)x^2 - 11(a+b+c)x - 3(a-b) = 0$ are

(a) real and equal (b) real and unequal

(c) purely imaginary (d) None of these

a b c

2. The graph of a quadratic polynomial $y = ax^2 + bx + c$; $a, b, c \in R$ is as shown.



Which one of the following is not correct?

(a) $b^2 - 4ac < 0$ (b) $\frac{c}{a} < 0$

(c) c is negative

(d) Abscissa corresponding to the vertex is $\left(-\frac{b}{2a}\right)$

y ordinate vertex < 0

$-\frac{D}{4a} < 0$, 3

$\frac{D}{4a} > 0$

$\boxed{a < 0}$

$y = c < 0$

$c < 0$

$\frac{D}{4a} > 0$ $a < 0$ $D < 0$

$b^2 - 4ac < 0$

$b^2 < 4ac$

$\frac{c}{a} > 0$

$-\frac{b}{2a} > 0$

$\frac{b}{a} < 0$

$\boxed{\begin{matrix} b > 0 \\ a < 0 \end{matrix}}$

$\boxed{\begin{matrix} a < 0 \\ b > 0 \\ c < 0 \end{matrix}}$

3. There is only one real value of 'a' for which the quadratic equation $ax^2 + (a+3)x + a - 3 = 0$ has two positive integral solutions. The product of these two solutions is

(a) 9 (b) 8 (c) 6 (d) 12

4. If for all real values of a one root of the equation $x^2 - 3ax + f(a) = 0$ is double of the other, $f(x)$ is equal to

(a) $2x$ (b) x^2 (c) $2x^2$ (d) $2\sqrt{x}$

$x^2 - 3ax + f(a) = 0$

$\alpha + 2\alpha = 3a$

$3\alpha = 3a$

$\boxed{\alpha = a}$

$\alpha \cdot 2\alpha = f(a)$

$f(a) = 2\alpha^2 = 2a^2$

$\boxed{f(x) = 2x^2}$

5. A quadratic equation the product of whose roots x_1 and

$\alpha + 2\alpha = 3a$

$3\alpha = 3a$

$\boxed{\alpha = a}$

$f(a) = 2\alpha = 2a$

$\boxed{f(x) = 2x^2}$

5. A quadratic equation the product of whose roots x_1 and x_2 is equal to 4 and satisfying the relation $\frac{x_1}{x_1 - 1} + \frac{x_2}{x_2 - 1} = 2$ is

(a) $x^2 - 2x + 4 = 0$ (b) $x^2 - 4x + 4 = 0$

(c) $x^2 + 2x + 4 = 0$ (d) $x^2 + 4x + 4 = 0$

6. If both roots of the quadratic equation $x^2 - 2ax + a^2 - 1 = 0$ lie in $(-2, 2)$, which one of the following can be $[a]$? (where $[\cdot]$ denotes the greatest integer function)

(a) -1 (b) 1 (c) 2 (d) 3

$ax^2 + bx + c = 0$

$-\left(\frac{b}{2(a)}\right)$

x ordinate ...

7. If $(-2, 7)$ is the highest point on the graph of $y = -2x^2 - 4ax + \lambda$, then λ equals

(a) 31 (b) 11 (c) -1 (d) $-\frac{1}{3}$

$\boxed{a = 2}$

$-\left(\frac{-4a}{\underset{= 2}{2(-2)}}\right) = -2$

$y = -2x^2 - 8x + \lambda \quad -(1)$

$y(-2, 7) \Rightarrow 7 = -2(-2)^2 - 8(-2) + \lambda$

$7 = -8 + 16 + \lambda$

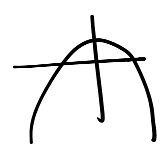
$\lambda = -1$

1,2 1,25 1 4

$4p - p^2 - 5 < 0$

$\alpha_1 < 1$

$\alpha_2 > 1$



8. If the roots of the quadratic equation $(4p - p^2 - 5)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unity, the number of integral values of p is

(a) 1 (b) 2 (c) 3 (d) 4

Graph is open down

$a < 0$

$f(1) > 0$

$4p - p^2 - 5 - 2p + 1 + 3p > 0$

$p^2 - 5p + 4 < 0$

$\boxed{1 < p < 4}$

$p = 2, 3$

9. Solution set of the equation $3^{2x^2} - 2 \cdot 3^{x^2 + x + 6} + 3^{2(x+6)} = 0$ is

(a) $\{-3, 2\}$ (b) $\{6, -1\}$ (c) $\{-2, 3\}$ (d) $\{1, -6\}$

10. Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$ $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root and $g(x) = ax^2 + px + q$ $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$, then

(a) α will be AM of the roots of $f(x) = 0$ and $g(x) = 0$

(b) α will be AM of the roots of $f(x) = 0$

(c) α will be AM of the roots of $f(x) = 0$ or $g(x) = 0$

(d) α will be AM of the roots of $g(x) = 0$

11. If x_1 and x_2 are the arithmetic and harmonic means of the roots of the equation $ax^2 + bx + c = 0$, the quadratic equation whose roots are x_1 and x_2, is

(a) $abx^2 + (b^2 + ac)x + bc = 0$

(b) $2abx^2 + (b^2 + 4ac)x + 2bc = 0$

(c) $2abx^2 + (b^2 + ac)x + bc = 0$

(d) None of the above

12. $f(x)$ is a cubic polynomial $x^3 + ax^2 + bx + c$ such that $f(x) = 0$ has three distinct integral roots and $f(g(x)) = 0$ does not have real roots, where $g(x) = x^2 + 2x - 5$, the minimum value of $a + b + c$ is

(a) 504 (b) 532 (c) 719 (d) 764

13. The value of the positive integer n for which the quadratic equation $\sum_{k=1}^{n} (x + k - 1)(x + k) = 10n$ has solutions α and $\alpha + 1$ for some α, is

(a) 7 (b) 11 (c) 17 (d) 25

(2)

$2^2 - \lambda 2 + 12 = 0$ $\boxed{\lambda = 8}$

14. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime, while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is

(a) 8 (b) 16 (c) 24 (d) 32

$\lambda^2 + 8\lambda + \mu = 0$

$x^2 + 8x + \mu = 0$

equal Roots

$D = 0$

$8^2 - 4.1.\mu = 0$

$64 = 4\mu$

$\boxed{\mu = 16}$

15. Number of real roots of the equation $\sqrt{x} + \sqrt{x - \sqrt{(1-x)}} = 1$ is

(a) 0 (b) 1 (c) 2 (d) 3

16. The value of $\sqrt{7 + \sqrt{7 - \sqrt{7 + \sqrt{7 - \ldots}}}}$ upto ∞ is

(a) 5 (b) 4

(c) 3 (d) 2

17. For any real x, the expression $2(k-x)\left[x+\sqrt{x^2+k^2}\right]$ cannot exceed

(a) k^2 (b) $2k^2$

(c) $3k^2$ (d) None of these

18. Given that, for all $x \in R$, the expression $\dfrac{x^2-2x+4}{x^2+2x+4}$ lies between $\dfrac{1}{3}$ and 3, the values between which the expression $\dfrac{9\cdot 3^{2x}+6\cdot 3^x+4}{9\cdot 3^{2x}-6\cdot 3^x+4}$ lies, are

(a) -3 and 1 (b) $\dfrac{3}{2}$ and 2

(c) -1 and 1 (d) 0 and 2

19. Let α, β, γ be the roots of the equation $(x-a)(x-b)(x-c)=d$, $d \neq 0$, the roots of the equation $(x-\alpha)(x-\beta)(x-\gamma)+d=0$ are

(a) a, b, d (b) b, c, d

(c) $\boxed{a, b, c}$ (d) $a+d, b+d, c+d$

$(x-a)(x-b)(x-c) = (x-\alpha)(x-\beta)(x-\gamma)+d$

a, b, c are the roots of the eq $(x-\alpha)(x-\beta)(x-\gamma)+d=0$

these are final roots are a, b, c

$x^2+ax+3=0$

$(\sqrt{x})^2+a(\sqrt{x})+3=0$

$\left(\frac{x}{4}\right)^2+a\left(\frac{x}{4}\right)+3=0$

$2x^2+3x+7=0$

$2(x-2)^2+3(x-2)+7=0$

98623

95723

151

all coefficients are not real

20. If one root of the equation $ix^2-2(1+i)x+2-i=0$ is $(3-i)$, where $i=\sqrt{-1}$, the other root is

(a) $3+i$ (b) $3+\sqrt{-1}$

(c) $-1+i$ (d) $\boxed{-1-i}$

α

$Sum = \dfrac{2(1+i)}{i}$

$\alpha+3-i = \dfrac{2(1+i)}{i} = \dfrac{2+2i}{i}$

$\alpha+3-i = 2-2i$

$\alpha = (-1-i)$

21. The number of solutions of $|[x]-2x|=4$, where $[x]$ denotes the greatest integer $\leq x$ is

(a) infinite (b) 4 (c) 3 (d) 2

$|[x]-2([x]+\{x\})|=4$

$|[x]+2\{x\}|=4$

22. If x^2+x+1 is a factor of ax^3+bx^2+cx+d, the real root of $ax^3+bx^2+cx+d=0$ is

(a) $-\dfrac{d}{a}$ (b) $\dfrac{d}{a}$ (c) $\dfrac{a}{d}$ (d) None of these

(a) $-\frac{d}{a}$ (b) $\frac{d}{a}$ (c) $\frac{a}{d}$ (d) None of these

$|[x] + 2\{x\}| = 4$

$2\{x\} = 0, 1$

23. The value of x which satisfy the equation

$$\sqrt{(5x^2 - 8x + 3)} - \sqrt{(5x^2 - 9x + 4)} = \sqrt{(2x^2 - 2x)} - \sqrt{(2x^2 - 3x + 1)}, \text{ is}$$

(a) 3 (b) 2
(c) 1 (d) 0

If $\{x\} = 0$ then $[x] = \pm 4$

$x = -4, 4$

$27 = 27 + 0$

$27.01 = 27 + 0.1$

$[x]\ \{x\}$

If $\{x\} = \frac{1}{2}$

$[x] + 1 = \pm 4$

$[x] = 3, -5$

$x = [x] + \{x\}$

$= 3.5, -4.5$

$x = -4, 4, -4.5, +3.5$

$2\times3\times3$ $2\times3 =$ Even $1\times3\times5 =$ odd

24. The roots of the equation

$$(a + \sqrt{b})^{x^2 - 15} + (a - \sqrt{b})^{x^2 - 15} = 2a$$

where $a^2 - b = 1$, are

(a) $\pm 2, \pm\sqrt{3}$ (b) $\pm 4, \pm\sqrt{14}$
(c) $\pm 3, \pm\sqrt{5}$ (d) $\pm 6, \pm\sqrt{20}$

25. The number of pairs (x, y) which will satisfy the equation

$x^2 - xy + y^2 = 4(x + y - 4)$, is

(a) 1 (b) 2
(c) 4 (d) None of these

26. The number of positive integral solutions of $x^4 - y^4 = \boxed{3789108}$ is

(a) 0 (b) 1 (c) 2 (d) 4

$x^4 - y^4 = 3789108$ ← Even

$x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)$ → All are Even

$x^4 - y^4 \to$ Even

or $x, y \to$ both even or

$x, y \to$ both odd

Here $(\text{even})^3 = 8$

$\frac{3789108}{8}$ ✗

$5 - 3 = 2$

$7 - 3 = 4$

$7 + 3 = 10$

there no integral soln

27. The value of 'a' for which the equation $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$, have a common root, is

(a) $a = 2$ (b) $a = -2$

(c) $a = 0$ (d) None of these

28. The necessary and sufficient condition for the equation $(1 - a^2)x^2 + 2ax - 1 = 0$ to have roots lying in the interval $(0, 1)$, is

(a) $a > 0$ (b) $a < 0$

(c) $a > 2$ (d) None of these

29. Solution set of $x - \sqrt{1 - |x|} < 0$, is

(a) $\left[-1, \frac{-1+\sqrt{5}}{2}\right)$ (b) $[-1, 1]$

(c) $\left[-1, \frac{-1+\sqrt{5}}{2}\right]$ (d) $\left(-1, \frac{-1+\sqrt{5}}{2}\right)$

30. If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0\ (b \neq c)$ have a common root, $a + 4b + 4c$, is equal to

(a) -2 (b) -1

(c) ~~0~~ (d) 1

$ax^2 + 2cx + b = 0$
$ax^2 + 2bx + c = 0$

$(a.2b - 2c.a)$
$(2c.c - b.2b) = (ba - ca)^2$

$4acc$

$2b(b-c) . 2(c^2 - b^2) = (ba - ca)^2$

$4a(c-b)(c+b) = a^2(b-c)$
$4a(c+b) = -a^2$
$4(c+b) = -a$
$a + 4b + 4c = 0$

31. If $0 < a < b < c$ and the roots α, β of the equation $ax^2 + bx + c = 0$ are non-real complex numbers, then

(a) $|\alpha| = |\beta|$ (b) $|\alpha| > 1$

(c) $|\beta| < 1$ (d) None of these

32. If A, G and H are the arithmetic mean, geometric mean and harmonic mean between unequal positive integers. Then, the equation $Ax^2 - |G|x - H = 0$ has

(a) both roots are fractions

(b) atleast one root which is negative fraction

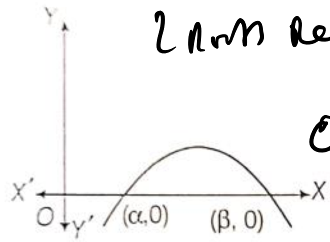
(c) exactly one positive root

(d) atleast one root which is an integer

$y = ax^2 + bx + c = 0$ $\quad y = a0^2 + b \cdot 0 + c = c \quad y < 0 \quad \boxed{c < 0}$

33. The adjoining graph of $y = ax^2 + bx + c$ shows that

2 roots real distinct $\quad \boxed{b^2 - 4ac > 0} \quad \boxed{a < 0}$



Cuts the y axis @ $x = 0$ $\quad y = c < 0 \quad \boxed{c < 0}$

x-coordinate > 0 $\quad -\frac{b}{2a} > 0 \quad \frac{b}{a} < 0 \quad a < 0 \quad b > 0$

(a) $a < 0$

(b) $b^2 < 4ac$

(c) $c > 0$

(d) a and b are of opposite signs

34. If the equation $ax^2 + bx + c = 0\ (a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then

(a) $b^2 - 4ac > 0$ (b) $c < 0$

(c) $a + |b| + c < 0$ (d) $4a + 2|b| + c < 0$

35. If $b^2 \geq 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all the roots of the equation will be real, if

(a) $b > 0, a < 0, c > 0$ (b) $b < 0, a > 0, c > 0$

(c) $b > 0, a > 0, c > 0$ (d) $b > 0, a < 0, c < 0$

35,

$\boxed{\frac{1}{r}, 1, r}$ Now $\frac{1}{r} + 1 + r = -b$

3 roots $\rightarrow \frac{a}{r}, a, ar \quad a > 0 \ r > 1$

Product of the roots $= 1$

$r + \frac{1}{r} > 2$

$\frac{1}{r} + r = -b - 1$

So, $-b - 1 > 2$

$b < -3$

$b \in (-\infty, -3)$

36. If roots of the equation $x^3 + bx^2 + cx - 1 = 0$ from an increasing GP, then

(a) $b + c = 0$

(b) $b \in (-\infty, -3)$

(c) one of the roots is 1

(d) one root is smaller than one and one root is more than one

Ans

$-\frac{b}{1} = \alpha + \beta + \gamma$

$\frac{c}{1} = \alpha\beta + \beta\gamma + \gamma\alpha$

$\frac{1}{1} = \alpha\beta\gamma \rightarrow \boxed{\frac{a}{r} \cdot a \cdot ar} = 1$

$a = 1$

then, $\frac{1}{r} \cdot 1 + 1 \cdot r + r \cdot \frac{1}{r} = c$

$\frac{1}{r} + r + 1 = c = -b$

$b + c = 0$

$\boxed{\frac{1}{r} < 1} \rightarrow$ 1st

2nd $\rightarrow 1$

3rd $\rightarrow r > 1$

37. Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R, a \neq 0$. Suppose $|f(x)| \leq 1, \forall\ x \in [0, 1]$, then

(a) $|a| \leq 8$ (b) $|b| \leq 8$

(c) $|c| \leq 1$ (d) $|a| + |b| + |c| \leq 17$

$x = 0, 1, \frac{1}{2}$

38. $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0$, $-1 < x < 0$, the value of $\sin 2\alpha$ is

(a) $\frac{24}{25}$ (b) $-\frac{12}{25}$

(c) $-\frac{24}{25}$ (d) $\frac{20}{25}$

$|c| \leq 1$

(a) $\frac{\ldots}{25}$ (b) $-\frac{\ldots}{25}$

(c) $-\frac{24}{25}$ (d) $\frac{20}{25}$

$\hookrightarrow x = 0, 1, \frac{1}{2}$

39. If $a, b, c \in R (a \neq 0)$ and $a + 2b + 4c = 0$, then equation $ax^2 + bx + c = 0$ has

(a) atleast one positive root
(b) atleast one non-integral root
(c) both integral roots
(d) no irrational root

$|c| \le 1$

$|a+b+c| \le 1$

$\left|\frac{a}{4} + \frac{b}{2} + c\right| \le 1$

$\boxed{-1 \le c \le 1}$

$-1 \le a+b+c \le 1$

$-4 \le a+2b+4c \le 4$

$-4 \le 4a+4b+4c \le 4$

$-4 \le -a-2b-4c \le 4$

After adding $-8 \le 3a+2b \le 8$

$-8 \le a+2b \le 8$

$\boxed{|b| \le 8}$

$-16 \le 2a \le 16$

$|2a| \le 16$

$|a| \le 8$

$|a|+|b|+|c| \le 17$

40. For which of the following graphs of the quadratic expression $f(x) = ax^2 + bx + c$, the product of abc is negative

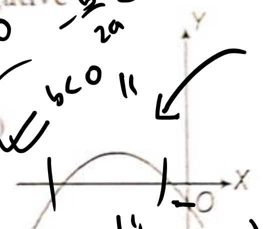
$x = 0$

$f(0) = c$

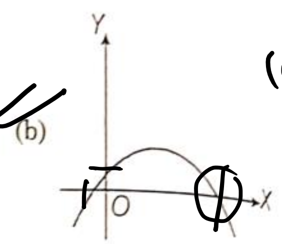
(a) $a<0,\ c<0$ $-\frac{b}{2a}<0$ $b<0$

$\boxed{abc<0}$

(a)



(b)



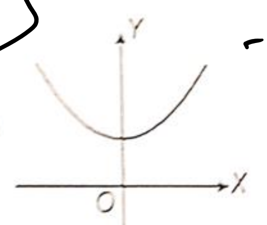
$-\frac{b}{2a} > 0$ $b>0$

$a<0$ $abc<0$

$c>0$

$\boxed{abc<0}$

(c)

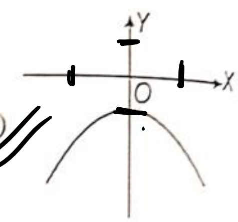


$-\frac{b}{2a} > 0$ $b<0$

$a>0$

$c>0$

(d)



$-\frac{b}{2a} < 0$ $a<0$ $b<0$

$c<0$ $\boxed{abc<0}$

41. If $a, b \in R$ and $ax^2 + bx + 6 = 0, a \neq 0$ does not have two distinct real roots, the

(a) minimum possible value of $3a + b$ is -2
(b) minimum possible value of $3a + b$ is 2
(c) minimum possible value of $6a + b$ is -1
(d) minimum possible value of $6a + b$ is 1

$D \le 0$ $f(x) \ge 0$

$f(3) \ge 0$

$9a + 3b + 6 \ge 0$

$3a + b \ge -2$

$f(6) \ge 0$

$36a + 6b + 6 \ge 0$

$6a + b \ge -1$

42. If $x^3 + 3x^2 - 9x + \lambda$ is of the form $(x - \alpha)^2 (x - \beta)$, then λ is equal to

α Double Root.

(a) 27 ~~(b) -27~~
(c) 5 ~~(d) -5~~

= 9061795123

$f'(x) = 0$ one Root $\to \alpha$

$3x^2 + 6x - 9 = 0 \to$ has one root α $(x+3)(x-1) = 0$ $x = 1, -3$

$x^2 + 2x - 3 = 0$

If $\alpha = 1$, $f(1) = 0$ gives $\lambda = 5$

$\alpha = -3$, $f(-3) = 0$ $\lambda = -27$

43. If $ax^2 + (b - \ldots$

If $\alpha = 1$, $f(1) = 0$ ...

$\alpha = -3$, $f(-3) = 0$ $\quad \lambda = -2$

43. If $ax^2 + (b-c)x + a - b - c = 0$ has unequal real roots for all $c \in R$, then

(a) $b < 0 < a$ (b) $a < 0 < b$

(c) $b < a < 0$ (d) $b > a > 0$

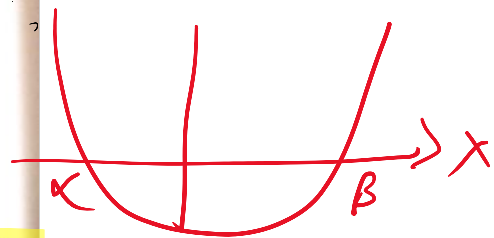
44. If the equation whose roots are the squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is identical with the given cubic equation, then

(a) $a = b = 0$

(b) $a = 0, b = 3$

(c) $a = b = 3$

(d) a, b are roots of $x^2 + x + 2 = 0$



45. If the equation $ax^2 + bx + c = 0\,(a > 0)$ has two real roots α and β such that $\alpha < -2$ and $\beta > 2$, which of the following statements is/are true?

(a) $4a - 2|b| + c < 0$

(b) $9a - 3|b| + c < 0$

(c) $a - |b| + c < 0$

(d) $c < 0,\ b^2 - 4ac > 0$

$b^2 - 4ac > 0$ $\quad b^2 > 4ac$ — (i)

$f(0) < 0$ $\quad c < 0$ — (ii) $\quad f(0) < 0$

$a + b + c < 0$ — (iii) $\quad f(-1) < 0$ $\quad a - b + c < 0$ — (iv)

$f(2) < 0$ $\quad 4a + 2b + c < 0$ — (v)

$f(-2) < 0$ $\quad 4a - 2b + c < 0$ — (vi)

$\boxed{a - |b| + c < 0}$ for (iii) & (iv)

$4a - 2|b| + c < 0$ for (v) & (vi)

67. The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is

3 Roots

68. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

$\rightarrow \dfrac{2\alpha\beta}{\alpha+\beta}$ $\qquad \dfrac{3\alpha\beta\gamma}{\alpha\beta+\alpha\gamma+\beta\gamma}$

69. If product of the real roots of the equation,

$$x^2 - ax + 30 = 2\sqrt{(x^2 - ax + 45)},\ a > 0,$$

is λ and minimum value of sum of roots of the equation is μ. The value of (μ) (where $(\cdot)$ denotes the least integer function) is

$N = \left(x+\frac{1}{x}\right)^6 - \left(x^3+\frac{1}{x^3}\right)^2$

$= \left(\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)\right)\left(\left(x+\frac{1}{x}\right)^3 - \left(x^3+\frac{1}{x^3}\right)\right)$

6 **70.** The minimum value of $\dfrac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left[\left(x+\frac{1}{x}\right)^3 + x^3 + \frac{1}{x^3}\right]}$ is $\quad 3\left(x+\frac{1}{x}\right)$

(for $x > 0$)

$\frac{N}{D} = 3\left(x+\frac{1}{x}\right)$

71. Let a, b, c, d are distinct real numbers and a, b are the roots of the quadratic equation $x^2 - 2cx - 5d = 0$. If c and

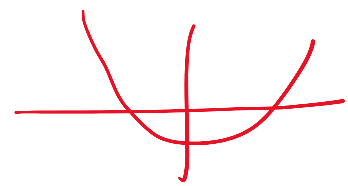
(for $x > 0$)

71. Let a, b, c, d are distinct real numbers and a, b are the roots of the quadratic equation $x^2 - 2cx - 5d = 0$. If c and d are the roots of the quadratic equation $x^2 - 2ax - 5b = 0$, the sum of the digits of numerical values of $a + b + c + d$ is

$\frac{N}{D}$, $N = D\left(3\left(x+\frac{1}{x}\right)\right) = 3\left(x+\frac{1}{x}\right)$, > 2, > 6

72. If the maximum and minimum values of $y = \dfrac{x^2 - 3x + c}{x^2 + 3x + c}$ are 7 and $\dfrac{1}{7}$ respectively, the value of c is

73. Number of solutions of the equation $\sqrt{x^2} - \sqrt{(x-1)^2} + \sqrt{(x-2)^2} = \sqrt{5}$ is



74. If α and β are the complex roots of the equation $(1+i)x^2 + (1-i)x - 2i = 0$, where $i = \sqrt{-1}$, the value of $|\alpha - \beta|^2$ is

75. If α, β be the roots of the equation $4x^2 - 16x + c = 0$, $c \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral values of c, are

$x^2 - 4x + \frac{c}{4} = 0$

(I) $D > 0$, $16 - c > 0$, $\boxed{c < 16}$

(II) $f(1) > 0$

$1 - 4 + \frac{c}{4} > 0$

$\frac{c}{4} > 3$

$\boxed{c > 12}$

76. Let r, s and t be the roots of the equation $8x^3 + 1001x + 2008 = 0$ and if $99\lambda = (r+s)^3 + (s+t)^3 + (t+r)^3$, the value of $[\lambda]$ is (where $[\cdot]$ denotes the greatest integer function)

(III) $f(3) > 0$

$9 - 12 + \frac{c}{4} > 0$

$\frac{c}{4} > 3$

$\boxed{c > 12}$

(IV) $f(2) < 0$

$4 - 8 + \frac{c}{4} < 0$

$\frac{c}{4} < 4$

$\boxed{c < 16}$

$\boxed{12 < c < 16}$

$c = 13, 14, 15$

88. For what values of m, the equation

$(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$ has ($m \in R$)

(i) both roots are imaginary? $D < 0$ $\quad 4m(m-3) < 0$ $\quad \boxed{0 < m < 3}$

(ii) both roots are equal? $D = 0$ $\quad m < 0, m > 3$

(iii) both roots are real and distinct? $D > 0$

d (iv) both roots are positive? Sum > 0 $\quad$ Prod > 0 $\quad D \geq 0$

e (v) both roots are negative? Sum < 0 $\quad$ Prod > 0 $\quad D \geq 0$

f (vi) roots are opposite in sign? Product < 0, $D > 0$

(vii) roots are equal in magnitude but opposite in sign? Sum $= 0$ $\quad D > 0$

(viii) atleast one root is positive? $d \cup f$

(ix) atleast one root is negative? $e \cup f$

(x) roots are in the ratio 2 : 3? $2\alpha, 3\alpha$

89. For what values of m, then equation
$2x^2 - 2(2m+1)x + m(m+1) = 0$ has $(m \in R)$

(i) both roots are smaller tha 2?

(ii) both roots are greater than 2?

(iii) both roots lie in the interval $(2, 3)$?

(iv) exactly one root lie in the interval $(2, 3)$?

(v) one root is smaller than 1 and the other root is greater than 1?

(vi) one root is greater than 3 and the other root is smaller than 2?

(vii) atleast one root lies in the interval $(2, 3)$?

(viii) atleast one root is greater than 2?

(ix) atleast one root is smaller than 2?

(x) roots α and β, such that both 2 and 3 lie between α and β?

90. If r is the ratio of the roots of the equation $ax^2 + bx + c = 0$, show that $\dfrac{(r+1)^2}{r} = \dfrac{b^2}{ac}$.

91. If the roots of the equation $\dfrac{1}{x+p} + \dfrac{1}{x+q} = \dfrac{1}{r}$ are equal in magnitude but opposite in sign, show that $p + q = 2r$ and that the product of the roots is equal to $\left(-\dfrac{p^2+q^2}{2}\right)$.

92. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the nth power of the other, then show that $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$.

93. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and γ, δ those of equation $lx^2 + mx + n = 0$, then find the equation whose roots are $\alpha\gamma + \beta\delta$ and $\alpha\delta + \beta\gamma$.

94. Show that the roots of the equation

$$(a^2 - bc)x^2 + 2(b^2 - ac)x + c^2 - ab = 0$$

are equal, if either $b = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$.

95. If the equation $x^2 - px + q = 0$ and $x^2 - ax + b = 0$ have a common root and the other root of the second equation is the reciprocal of the other root of the first, then prove that $(q - b)^2 = bq(p - a)^2$.

96. If the equation $x^2 - 2px + q = 0$ has two equal roots, then the equation $(1+y)x^2 - 2(p+y)x + (q+y) = 0$ will have its roots real and distinct only, when y is negative and p is not unity.

97. Solve the equation $x^{\log_x (x+3)^2} = 16$.

98. Solve the equation

$$(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \frac{101}{10(2-\sqrt{3})}.$$

99. Solve the equation $x^2 + \left(\frac{x}{x-1}\right)^2 = 8.$

100. Solve the equation

$$\sqrt{(x+8)+2\sqrt{(x+7)}} + \sqrt{(x+1)-\sqrt{(x+7)}} = 4.$$

101. Find all values of a for which the inequation

$4^{x^2} + 2(2a+1)2^{x^2} + 4a^2 - 3 > 0$ is satisfied for any x.

102. Solve the inequation $\log_{x^2+2x-3}\left(\dfrac{|x+4|-|x|}{x-1}\right) > 0.$

103. Solve the system $|x^2 - 2x| + y = 1, x^2 + |y| = 1.$

104. If α, β, γ are the roots of the cubic $x^3 - px^2 + qx - r = 0.$
Find the equations whose roots are

(i) $\beta\gamma + \dfrac{1}{\alpha}, \gamma\alpha + \dfrac{1}{\beta}, \alpha\beta + \dfrac{1}{\gamma}$

(ii) $(\beta + \gamma - \alpha), (\gamma + \alpha - \beta), (\alpha + \beta - \gamma)$

Also, find the value of $(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)(\alpha + \beta - \gamma).$

105. If $A_1, A_2, A_3, \ldots, A_n, a_1, a_2, a_3, \ldots, a_n, a, b, c \in R$, show that the roots of the equation

$$\frac{A_1^2}{x-a_1}+\frac{A_2^2}{x-a_2}+\frac{A_3^2}{x-a_3}+\ldots+\frac{A_n^2}{x-a_n}$$
$$= ab^2 + c^2 x + ac \text{ are real.}$$

106. For what values of the parameter a the equation $x^4 + 2ax^3 + x^2 + 2ax + 1 = 0$ has atleast two distinct negative roots?

107. If $[x]$ is the integral part of a real number x. Then solve $[2x] - [x+1] = 2x$.

108. Prove that for any value of a, the inequation $(a^2 + 3)x^2 + (a+2)x - 6 < 0$ is true for atleast one negative x.

109. How many real solutions of the equation $6x^2 - 77[x] + 147 = 0$, where $[x]$ is the integral part of x ?

110. If α, β are the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ and γ, δ are the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$, such that $\alpha, \beta \in (\gamma, \delta)$, find the value of '$a$'.

111. If the equation $x^4 + px^3 + qx^2 + rx + 5 = 0$ has four positive real roots, find the minimum value of pr.