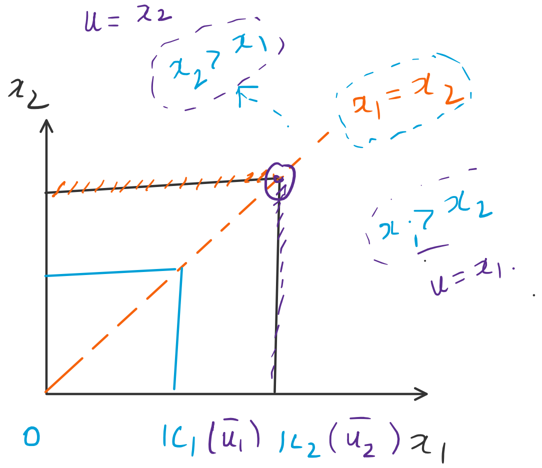
Q. $u = \max\{x_1, x_2\}$

$$
\begin{cases}
u = x_1, & x_1 > x_2. \\
u = x_2, & x_2 > x_1. \\
u = x_1 = x_2, & x_1 = x_2.
\end{cases}
$$



Slope of IC $= \dfrac{dx_2}{dx_1}$

(i): $x_2 > x_1 \quad u = x_2$

Diff: $du = dx_2$

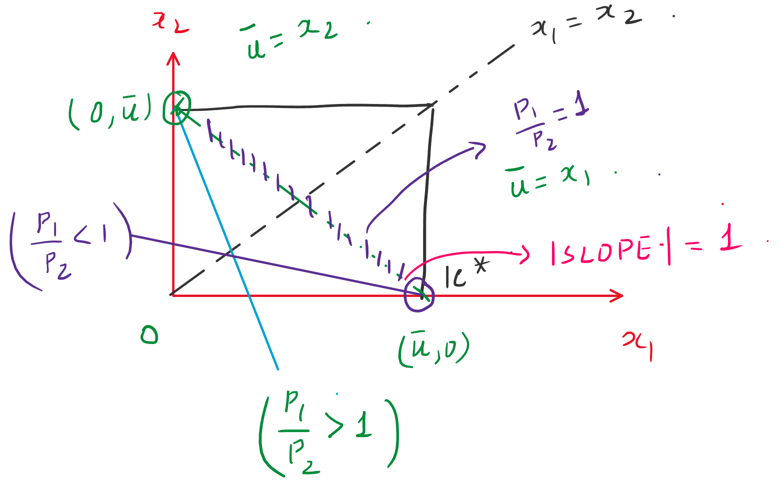
For IC, $du = 0 \Rightarrow dx_2 = 0$.

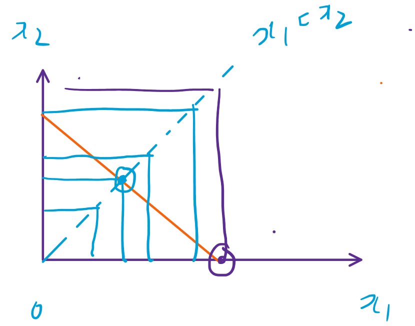
$\Rightarrow \dfrac{dx_2}{dx_1} = 0$.

(ii) $x_1 > x_2$: $u = x_1$

Diff: $du = dx_1$

For IC: $du = 0 \Rightarrow dx_1 = 0 \Rightarrow \left.\dfrac{dx_2}{dx_1}\right|_{IC} \to \infty$.





B.L: $M = P_1x_1 + P_2x_2$

(i) $\dfrac{P_1}{P_2} > 1 \Rightarrow \boxed{P_1 > P_2}$ then $x_1^* = 0$, $x_2^* = \dfrac{M}{P_2}$.

(ii) $\dfrac{P_1}{P_2} < 1 \Rightarrow \boxed{P_1 < P_2}$, then $x_1^* = \dfrac{M}{P_1}$, $x_2^* = 0$.

Buy the good which is cheaper.

(iii) $\dfrac{P_1}{P_2} = 1 \Rightarrow P_1 = P_2$ then either $\left(0, \dfrac{M}{P_2}\right)$ or $\left(\dfrac{M}{P_1}, 0\right)$.

Non-linear in Good 1   Linear in Good 2

[Non-linear Good → No income ...

Q. $u = \ln x_1 + x_2$ ---- (Quasi linear preference) x_2

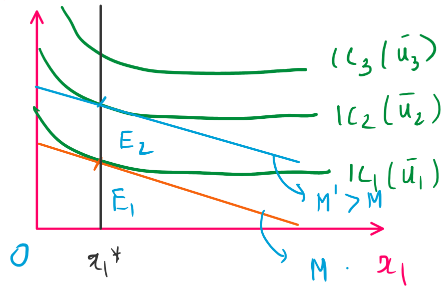
Non-linear in Good 1

Linear in Good 2

[Non-linear Good ⇒ No income effect]

$$MRS = \frac{MU_1}{MU_2} = \frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \frac{1/x_1}{1} = \frac{1}{x_1}.$$

$$\frac{\partial MRS}{\partial x_1} = -\frac{1}{x_1^2} < 0 \Rightarrow \text{convex IC's}.$$



At opt: $MRS = \frac{P_1}{P_2} \Rightarrow \frac{1}{x_1} = \frac{P_1}{P_2} \Rightarrow P_2 = P_1 x_1 \Rightarrow \boxed{x_1^* = \frac{P_2}{P_1}}$

BL: $M = P_1 x_1 + P_2 x_2$.

$M = P_2 + P_2 x_2 \Rightarrow M - P_2 = P_2 x_2 \Rightarrow x_2^* = \frac{M - P_2}{P_2}$

$= \left(\frac{M}{P_2} - 1\right)$

$\boxed{x_1^* = \frac{P_2}{P_1}}$ No income effect , $x_2^* = \left(\frac{M}{P_2} - 1\right)$ --- Marshallian demand curves for Good 1 & Good 2.

Note: Max $u = u(x_1, x_2)$ s.t $M = P_1 x_1 + P_2 x_2$.

Parameters: M, P_1, P_2.

Variables: x_1, x_2.

If $x_2^* < 0$

$\Rightarrow \left(\frac{M}{P_2} - 1\right) < 0$.

$\Rightarrow \frac{M}{P_2} < 1 \Rightarrow \boxed{M < P_2}$

Solving this we get the Marshallian demand curves:

$\boxed{x_1^* = x_1^*(M, P_1, P_2)}$ $\qquad x_2^* = x_2^*(M, P_1, P_2)$.

$M \uparrow \Rightarrow x_1^* \uparrow \Rightarrow$ Income Effect.

$P_1 \uparrow \Rightarrow x_1^* \downarrow \Rightarrow$ Own price Effect.

Note: $u = u(x_1, x_2)$.

$MU_1 = \frac{\partial u}{\partial x_1} > 0 \Rightarrow [x_1 \uparrow \Rightarrow u \uparrow]$; If $MU_1 < 0$ $[x_1 \uparrow \Rightarrow u \downarrow]$

↳ x_1 becomes a "bad"

$MU_2 = \frac{\partial u}{\partial x_2} > 0 \Rightarrow [x_2 \uparrow \Rightarrow u \uparrow]$; Then, $\boxed{x_1^* = 0}$

$$MU_2 = \frac{\partial u}{\partial x_2} > 0 \Rightarrow [x_2 \uparrow \Rightarrow u \uparrow] \; ; \quad \text{Then, } \boxed{x_1^* = 0}$$

∴ If the marginal utility turns -ve, then the individual will not consume any extra units of the good.

B. $u = -\left[(x_1 - 10)^2 + (x_2 - 20)^2\right]$.

$$\frac{\partial u}{\partial x_1} = -2(x_1 - 10) \; (>0 \text{ if } x_1 < 10) \quad \boxed{(<0 \text{ if } x_1 > 10)}$$

$$\frac{\partial u}{\partial x_2} = -2(x_2 - 20) \; (>0 \text{ if } x_2 < 20) \quad \boxed{(<0 \text{ if } x_2 > 20)}$$

At opt: $x_1^* = 10$, $x_2^* = 20$