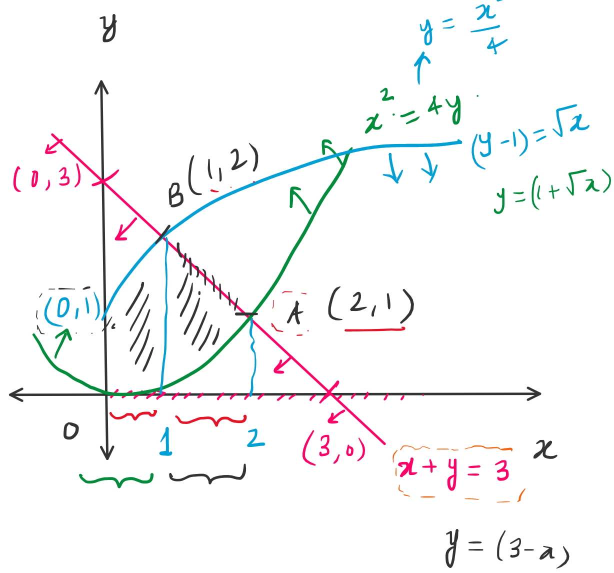
27 February 2024 21:07

Q. The area of the region :

$\{(x,y) : x \geq 0,\ x+y \leq 3,\ x^2 \leq 4y$ and $y \leq 1+\sqrt{x}\}$ is:

(a) $5/2$ (b) $59/12$ (c) $3/2$ (d) $7/3$.



$y = 1+\sqrt{x}$.

or, $(y-1) = \sqrt{x} \Rightarrow x=0,\ y=1$

$y = \sqrt{x}$,

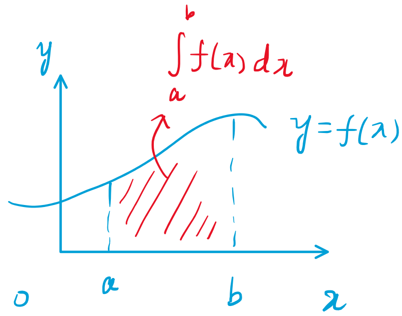
HW

$$A = \int_0^1 (-)\,dx + \int_1^2 (-)\,dx$$

$$= \int_0^1 (1+\sqrt{x}) - \frac{x^2}{4}\,dx + \int_1^2 (3-x) - \frac{x^2}{4}\,dx = 5/2 \text{ (check!)}$$

$$= \int_0^1 (1+\sqrt{x})\,dx + \int_1^2 (3-x)\,dx - \int_0^2 \frac{x^2}{4}\,dx$$

Graphical Interpretation: $\int_a^b f(x)\,dx$ = area under the curve $y = f(x)$.



Let $u = f(x,y)$.

$$\int_a^b \int_c^d f(x,y)\,dx^{(I)}\,dy^{(II)} = \text{Double Integral}.$$

order of integration.

y x

Eg: $\int_0^1 \int_1^2 (xy)\,dx^{(I)}\,dy^{(II)}$. $1 \leq x \leq 2$. $0 \leq y \leq 1$.

$$= \int_0^1 \left\{ \int_1^2 xy\,dx \right\} dy$$

[Integrate (xy) w.r.t x by treating 'y' as constant].

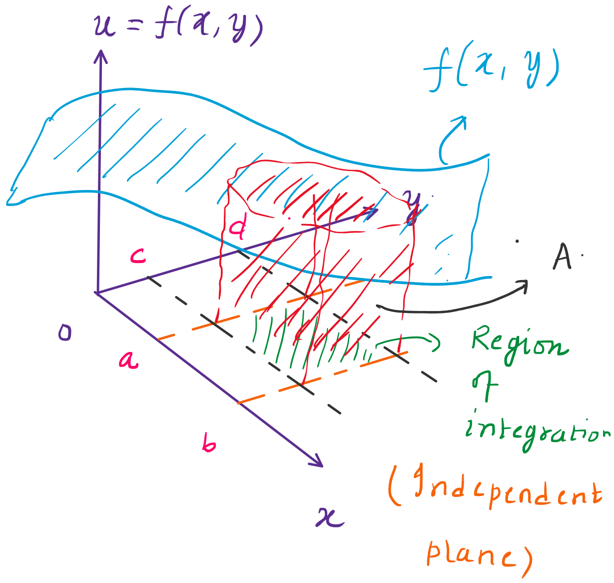
$$= \int_0^1 \left(\int_1^2 xy \, dx \right) dy \quad$$ [Integrate (xy) w.r.t x by treating 'y' as constant].

$$= \int_0^1 y \cdot \left[\frac{x^2}{2}\right]_1^2 dy = \int_0^1 y\left(2 - \frac{1}{2}\right) dy$$

$$= \frac{3}{2}\int_0^1 y \, dy = \frac{3}{2}\left[\frac{y^2}{2}\right]_0^1 = \frac{3}{2}\cdot\frac{1}{2} = \frac{3}{4}.$$

Eg: Graphical Interpretation of:

$$A = \int_{x=a}^{b}\int_{y=c}^{d} f(x,y)\, dy\, dx.$$

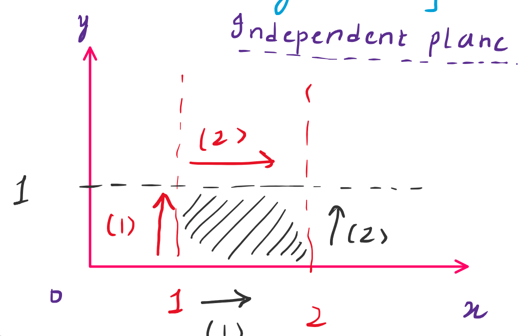


$x \in [a, b]$, $y \in [c, d]$.

Eg: $\displaystyle\int_0^1 \int_1^2 (xy)\, \underset{(\mathrm{I})}{dx}\, \underset{(\mathrm{II})}{dy}$.
$\boxed{\begin{array}{c} 1 \le x \le 2 \\ 0 \le y \le 1 \end{array}}$

$\displaystyle\int_1^2 \int_0^1 xy \, dy\, dx \ldots$ [Reversed the order of integration].

$$\int_1^2 \left\{ \int_0^1 xy \, dy \right\} dx = 3/4.$$

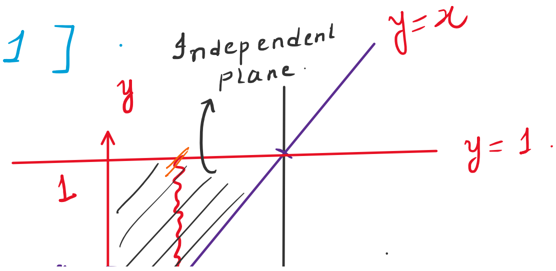


(*) Note: When range of x, y are independent of each other, the reversing the order of integration does not change the value.

Q. $\displaystyle\int_0^1 \int_x^1 f(x,y)\, dy\, dx$.

$y \in [x, 1]$ — dependent range.
$x \in [0, 1]$.

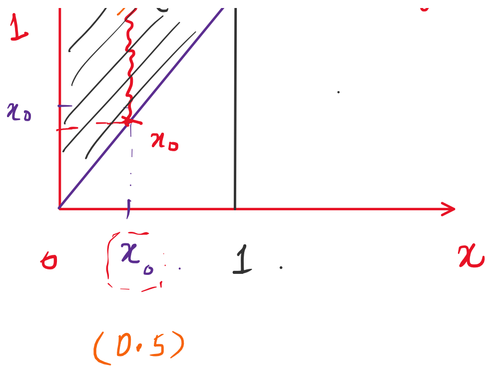
$y \in [x, 1]$.



$x = x \Rightarrow \min y = x$ (o.r)

$x = x_0 \Rightarrow \min y = x_0 \ (0.5)$

$\max y = 1.$



$$\int_0^1 \int_x^1 f(x,y)\,dy\,dx = \int_0^1 \int_0^y f(x,y)\,dx\,dy.$$

$x \in [0,1] \Rightarrow$ Independent

$y \in [x, 1].$