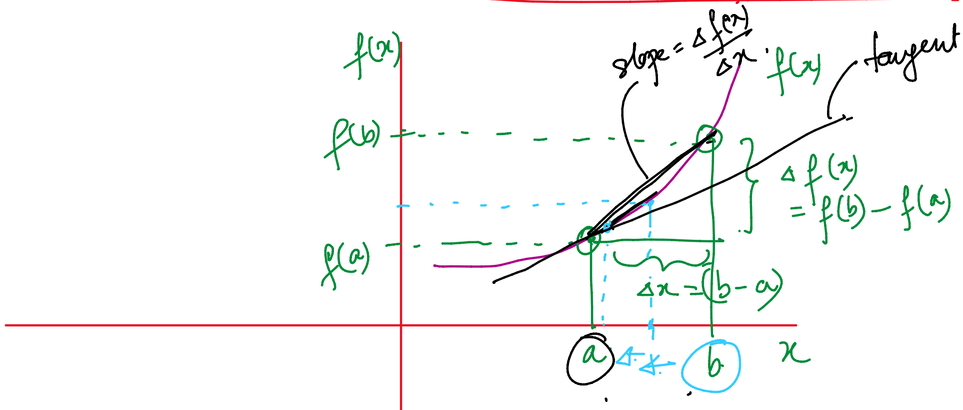
18 November 2023 17:01

Fundamentals of derivatives



Instantaneous rate of change at a particular value of x

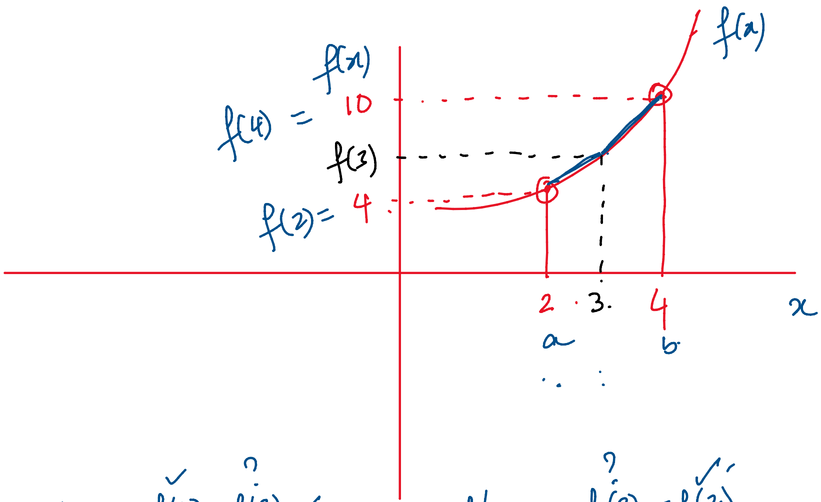
$$\text{Rate of change} = \frac{\Delta f(x)}{\Delta x} = \frac{f(b)-f(a)}{(b-a)}$$

$$\lim_{b \to a} \frac{f(b)-f(a)}{(b-a)} = f'(a)$$

$\downarrow$

derivative of $f(x)$ at $x = a$.

Slope of tangent at a = instantaneous rate of change at $x = a$.



$\boxed{f'(3) = ?}$

Approximation using the concept of derivative

$$f'(a) \approx \frac{f(b)-f(a)}{(b-a)}$$

$\Downarrow$

$b - a \approx 0$.

$$f'(3) \approx \frac{f(4)-f(3)}{4-3}$$

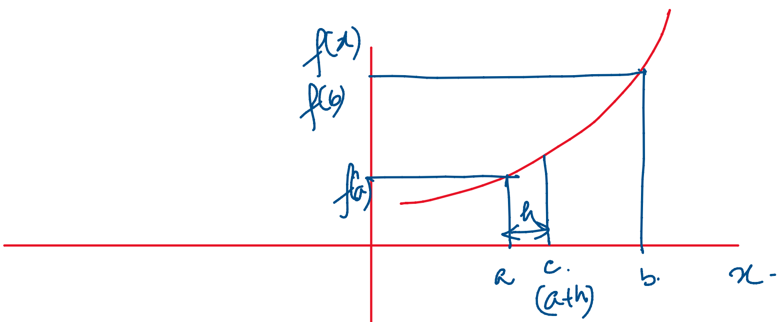
$$f'(2) \approx \frac{f(3)-f(2)}{3-2}$$

$f'(2) \approx f'(3)$

$f(4) - f(3) \approx f(3) - f(2)$

$2f(3) \approx f(2) + f(4)$

$$\boxed{f(3) = \frac{f(2)+f(4)}{2}}$$



$$f'(a) \approx \frac{f(a+h)-f(a)}{h}$$

$h f'(a) \approx f(a+h) - f(a)$

$$\boxed{f(a+h) \approx f(a) + h f'(a)}$$

$a = 2$

$f(2) = 4$

$f'(2) = 0.5$

$f'(3) = ?$

$f(3) \approx f(2) + 1 \times 0.5 = 4.5$

$a=2$ $\quad f(3)$:

$f(2)=4$

$f'(2)=0.5$

$b=4$

$f(4)=10.$

$f(3) \approx f(2) + 1 \times 0.5 = 4.5$

$f'(3) \approx \dfrac{f(4)-f(3)}{4-3} = 10-4.5 = 5.5$

$2f(x) + f\left(\frac{1}{x}\right) = 3x.$ $\quad$ (a = $f(x)$, b = $f(\frac{1}{x})$)

find $f'(2) = ?$

$$\downarrow$$

find $f(x)$

$\boxed{2a+b = 3x}$ ①

equn in 2 variables a, b

Trick: $\boxed{\text{substitute } x \text{ with } \tfrac{1}{x}}$

$2f\left(\frac{1}{x}\right) + f(x) = 3 \cdot \frac{1}{x}.$

$\boxed{2b + a = \frac{3}{x}}$ ②

$4a + 2b = 6x.$

$3a = 6x - \frac{3}{x}.$

$\boxed{f(x) = a = 2x - \frac{1}{x}.}$

$f'(x^n) = nx^{n-1}$

$f'(x) = 2 - (-1)x^{-2} = 2 + \frac{1}{x^2}$

$3f(x) + f(4-x) = x+2.$

replace x with $4-x$.

$4-x \rightarrow 4-(4-x)$
$= x.$

$3f(4-x) + f(x) = 4-x+2 = 6-x$

$3f(4-x) + 9f(x) = 3x+6$

$8f(x) = 4x$

$\boxed{f(x) = \frac{x}{2}}$

$f(x) = 3x^3 + 4x.$

Is $f(x)$ bijective?

one-one $\quad$ onto

$\boxed{f'(x) = 9x^2 + 4}$ ≥ 0

+ve

Condition for $f(x)$ to be bijective is that $f(x)$ should be strictly increasing or strictly decreasing

$f'(x) > 0$ or $f'(x) < 0$

$f(x) = ax^n + bx^{n-2} + cx^{n-4} + \dots$

where n is ODD

$a, b, c, \dots \geq 0$

$f(x) = 3x^3 - 4x$. $\quad f'(x) = 9x^2 - 4$

$9x^2 - 4 > 0 \Rightarrow x^2 > \frac{4}{9} \Rightarrow x > \frac{2}{3}$ or $x < -\frac{2}{3}$

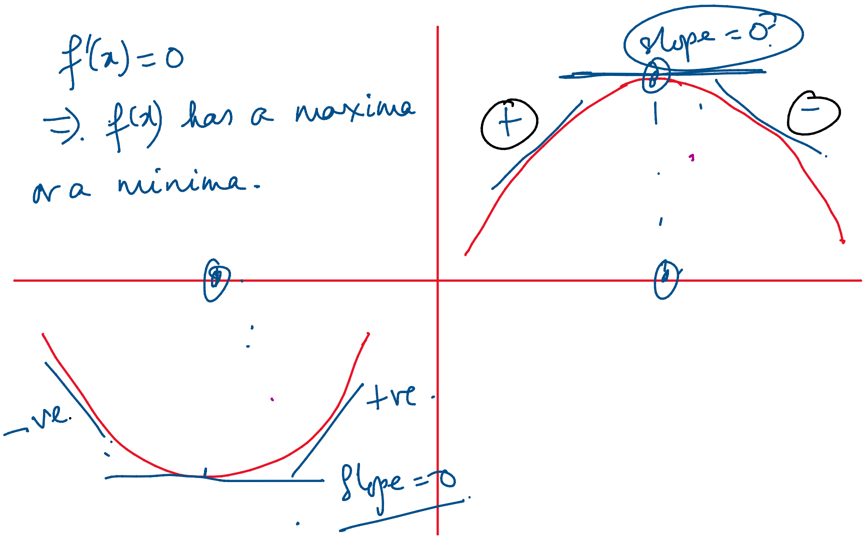
$\frac{u'v - v'u}{v^2}$

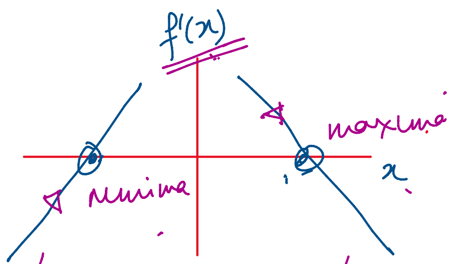
$f(x) = \frac{x+1}{x-1}$ $\quad f'(x) = \frac{(x-1) - (x+1)}{(x-1)^2}$

$= \frac{-2}{(x-1)^2} < 0 \qquad (x-1)^2 > 0$

$f'(x) = 0$

$\Rightarrow f(x)$ has a maxima or a minima.





$[f'(x)]' > 0 \qquad [f'(x)]' < 0$

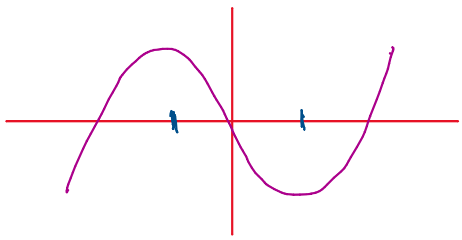
$f''(x) > 0 \qquad f''(x) < 0$

$f(x) = 3x^3 - 4x$.

$f'(x) = 9x^2 - 4 \qquad 9x^2 - 4 = 0 \qquad \boxed{x = \pm \frac{2}{3}}$

$f''(x) = 18x$.

$x = -2/3$. $\quad f''(x) < 0$. MAXIMA $\qquad x = 2/3 \quad f''(x) > 0$ MINIMA.



x	0	2	4	6
$f(x)$	6	10	24	50
$f'(x)$	1	3		

$f'(2) = \frac{f(3) - f(2)}{3-2}$

$f'(0) = \frac{f(1) - f(0)}{1-0}$

$1 = f(1) - 6$

$f(1) = 7$

$f'(1) = \frac{f(2) - f(1)}{2-1} = 10-7$

$= 3$