20 January 2024 14:24

Periodic functions.

$f(x+T) = f(x)$ ⟵

↓

Period.

ILLUSTRATION 3 *If a periodic function $f(x)$ satisfies the equation* $f(x+1) + f(x-1) = \sqrt{3}\, f(x)$ *for all* $x \in R$.
Then, period of $f(x)$ is

(a) 2 (b) 6 (c) 12 (d) 4

$x \to x+1$ $\quad f(x+2) + f(x) = \sqrt{3}\, f(x+1)$

$x \to x-1$ $\quad f(x) + f(x-2) = \sqrt{3}\, f(x-1)$

$2f(x) + f(x+2) + f(x-2) = \sqrt{3}\left(f(x+1) + f(x-1)\right) = \sqrt{3}\sqrt{3} f(x)$

$= 3f(x)$

$f(x) = f(x+2) + f(x-2)$

$x \to x+2$ $\quad f(x+2) = f(x+4) + f(x)$

$f(x) + f(x+2) = f(x+2) + f(x-2) + f(x+4) + f(x)$

$f(x+4) + f(x-2) = 0.$

$f(x-2) = -f(x+4)$

$x \to x+2$ $\quad f(x) = -f(x+6)$ $\quad f(x+6) = -f(x)$

$x \to x+6.$ $\quad f(x+6) = -f(x+12)$

$-f(x) = -f(x+12)$

$f(x) = f(x+12)$ $\quad T = 12$

ILLUSTRATION 5 *The period of the function*

$f(x) = |\sin x| - |\cos x|$, *is*

(a) $\pi/2$ (b) π (c) 2π (d) *none of these*

$f(x) = f(x+T)$

$= |\sin(x+T)| - |\cos(x+T)|$

$|\sin(x+\pi)| - |\cos(x+\pi)|$

$= |\sin x| - |\cos x|$

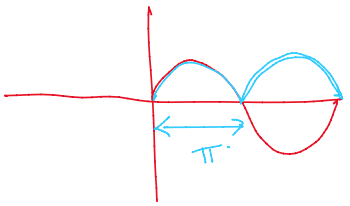


ILLUSTRATION 6 *The period of the function*

$f(x) = \big|\,|\sin x| - |\cos x|\,\big|$, *is*

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) *none of these*

$f(x) = \sin x - \cos x$.

$T = 2\pi$.

$f(x) = |\sin x| - |\cos x|$

$T = \pi$.

$f(x+\pi/2) = \big|\,|\sin(x+\pi/2)| - |\cos(x+\pi/2)|\,\big|$

$= \big|\,|\cos(-x)| - |\sin(-x)|\,\big|$

$= \big|\,|\cos x| - |\sin x|\,\big|$

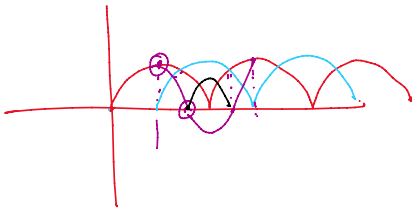


ILLUSTRATION 7 *If the function* $f(x) = \sin x + \cos ax$ *is periodic, then*

(a) $a \in Z$ (b) $a \in N$ (c) $a \in Q$ (d) $a \in R$

$\sin x = \sin(x + 2n\pi)$ $\cos x = \cos(x + 2n\pi)$

$a\sin x + b\cos x$

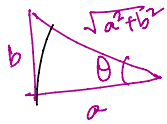
$= \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x\right]$

$\sin x + \cos x \rightarrow \sin(x+2n\pi) + \cos(x+2n\pi)$

$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right) \sim \sin x$

$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ $\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$

$\sin x + \cos x = \sin x + \sin\left(\frac{\pi}{2} - x\right)$



$\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ $\quad \sin\theta = \frac{b}{\sqrt{a^2+b^2}}$

$= \sqrt{a^2+b^2}\,\sin(x+\theta)$

$\sin x + \cos x = \sin x + \sin\left(\frac{\pi}{2} - x\right)$

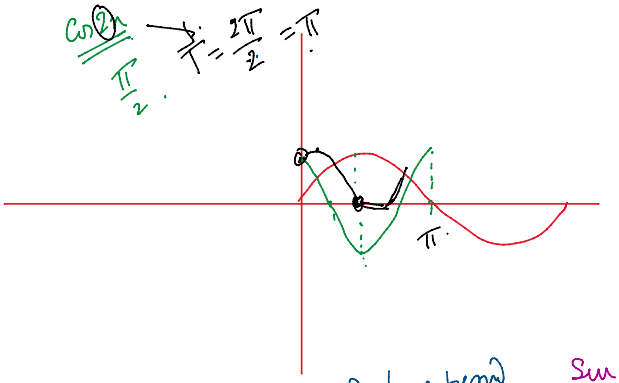
$= 2\sin\frac{\pi}{4}\cos\left(x - \frac{\pi}{4}\right)$

$= 2\sin\frac{\pi}{4}\cos\left(\frac{\pi}{4} - x\right)$

$2 \cdot \frac{1}{\sqrt{2}}\sin\left[\frac{\pi}{2} - \left(\frac{\pi}{4} - x\right)\right]$

$\cos(\theta) = \cos(-\theta)$

$\sin x + \cos 2x.$



$\sin x \rightarrow$ Time period $= 2\pi$.

$\cos ax \rightarrow$ Time period $= \frac{2\pi}{a}$.

$m\frac{2\pi}{a} = n\,2\pi$.

$a = \frac{m}{n}$

$\cos ax \rightarrow T = \frac{2\pi}{a}$.

$m\frac{2\pi}{a} = n\,2\pi$

$a = \frac{m}{n} \rightarrow$ Rational

Combined time period $= \frac{\text{LCM (Nelm)}}{\text{HCF (den)}}$

$\sin ax + \cos bx.$

$T = \frac{2\pi}{a}$ $\quad T = \frac{2\pi}{b}$.