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Poisson Distribution

X ~ discrete random variable

↳ follows probability mass func (pmf).

what is pmf of poisson Distribution?

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

where $x = 0, 1, 2, \ldots \infty$.

and λ is the parameter of the distribution.

$E(x) = \sum_{x=0}^{\infty} x \cdot f(x) = \sum x \frac{e^{-\lambda}\lambda^x}{x!}$

Properties:

Mean $(\mu) = E(x) = \lambda$ ✓

Variance $\mu_2 = V(x) = \lambda$ ✓

$V(x) = E(x^2) - \{E(x)\}^2$

$= E(x(x-1)+x) - \{E(x)\}^2$

$= \boxed{E(x(x-1))} + E(x) - \{E(x)\}^2$

$E(x(x-1)) = \sum x(x-1) f(x)$

$= \sum x(x-1) \frac{e^{-\lambda}\lambda^x}{x!}$

In poisson distribution, mean = variance = λ

$\therefore$ S.D $= \sqrt{\lambda}$

Mode : The distribution is bimodal when λ is an integer, the modes being λ and $\lambda - 1$

Distribution is fractional, then distribution is unimodal.

Skewness, $\gamma_1 = \sqrt{\beta_1} = \sqrt{\frac{\mu_3^2}{\mu_2^3}} = \frac{1}{\sqrt{\lambda}} > 0 \Rightarrow$ P.D is +vely skewed.

kurtosis, $\gamma_2 = \beta_2 - 3 = \left(3 + \frac{1}{\lambda}\right) - 3 = \frac{1}{\lambda} > 0 \Rightarrow$ P.D is leptokurtic.

Numericals

Q1. If a random variable X follows Poisson Distribution satisfying $2 \cdot P(X=0) = P(X=1)$, determine $P(X>0)$, $P(X>0 \mid X<2)$ and $E(X)$. Given, $e^{-2} = 0.1353$.

Let λ be the parameter of the distribution.

Then $P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$, $x = 0,1,2,\cdots\infty$

Given condition $2P(X=0) = P(X=1)$

$$2 \cdot \frac{e^{-\lambda}\lambda^{0}}{0!} = \frac{e^{-\lambda}\lambda^{1}}{1!}$$

$$2 \cdot e^{-\lambda} = e^{-\lambda} \cdot \lambda$$

$X = 0 \to \infty$

$$\therefore \boxed{\lambda = 2} = \text{mean}.$$

$\therefore P(X>0) = 1 - P(X=0)$

$= 1 - \dfrac{e^{-\lambda}\lambda^{0}}{0!}$

$= 1 - e^{-2}$

$= 1 - 0.1353$

$= 0.8647$ (ans)

$P(X>0 \mid X<2)$

$= \dfrac{P[(X>0) \cap (X<2)]}{P(X<2)}$

$= \dfrac{P(X=1)}{P(X=0) + P(X=1)}$

$= \dfrac{e^{-2}\, 2^{1}/1!}{e^{-2} + e^{-2} \cdot}$

$$= \frac{e^{-2}\, 2/1!}{e^{-2} + e^{-2}.2}$$

$$= \frac{2.\cancel{e^{-2}}}{\cancel{e^{-2}}(1+2)}$$

$$= \frac{2}{3} \text{ (ans).}$$

Mean, $E(x) = \Sigma x \cdot f(x)$

$$V(x) = E\left(x - E(x)\right)^2$$
$$= E(x^2) - \left(E(x)\right)^2$$

Mean, $E(x) = \sum_{x} x \cdot f(x)$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \sum_{x=0}^{\infty} \cancel{x} e^{-2} \frac{2^{x}}{x!}$$

$x! = \cancel{x}(x-1)!$

$$= e^{-2} \sum_{x=1}^{\infty} \frac{2^{x}}{(x-1)!}$$

$$= 2e^{-2} \sum_{x=1}^{\infty} \frac{2^{x-1}}{(x-1)!}$$

$$= 2e^{-2} \cdot e^{2} = 2$$

Note:
We know in P.D mean $= \lambda = 2$

2) The manufacturer of a certain electronic component knows that 3% of his product is defective. He sells the components in boxes of 100 and guarantees that not more than 3% in any box will be defective. What is the probability that a

guarantees that not more than 3%
will be defective. What is the probability that a
box will fail to meet the
guarantee? (Given $e^3 = 20.1$).

$n \rightarrow$ no. of box 100 $\quad$ $X \Rightarrow$ no. of defectives. $\quad p = 3\% = 0.03$

mean $= np = \lambda = 100 \times 0.03 = 3$

$\therefore$ PD with $\lambda = 3$ will be $P(X=x) = \dfrac{e^{-3} 3^x}{x!}$

Probability that a box fails to meet gurantee.

$$P(X > 3) = 1 - P(X \leq 3)$$

$$= 1 - \sum_{x=0}^{3} \frac{e^{-3} 3^x}{x!}$$

$$= 1 - e^{-3}\left[\frac{3^0}{0!} + \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right]$$

$$= 1 - e^{-3}\left[\underbrace{1 + 3 + \frac{9}{2} + \frac{27}{6}}\right]$$

$$= 1 - e^{-3} \times (13)$$

$$= 1 - 13e^{-3}$$

$$= 1 - \frac{13}{20.1} \qquad \left[\because e^3 = 20.1 \quad \therefore e^{-3} = \frac{1}{20.1}\right]$$

$$= 1 - 0.647$$

$$= 1 - 0.647^{20.1}$$

$$= 0.353 \text{ (ans)}$$

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