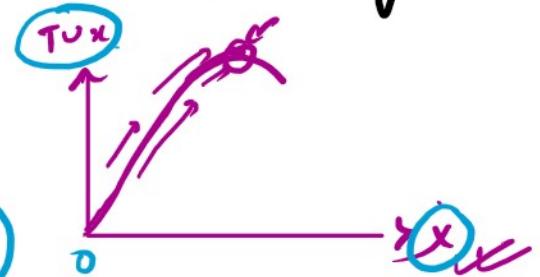


Consumer behaviour - Utility Analysis.

Cardinal Approach:

1. Utility is cardinally measurable \rightarrow that means, utility is measurable by both principle and practical by various cardinal numbers (in terms of money).
2. Marginal utility of money is constant.
i.e., the addition utility derived from the additional unit of money is constant.
That is why utility derived from the commodity is constant for additional unit.
3. Diminishing MU is operated for more and more commodity consumed.
that is if the consumer consumes more & more commodity, then marginal utility decreases.
There are 2 reasons behind it
 (i) consumer may be saturated
 (or reached the point of satiation)
 (ii) goods may have substitutes.
4. Utility derived from a commodity is only dependent on the consumption units of that commodity and not any other commodity or a consumption.
5. The consumer is rational.
6. The money income of the consumer and the consumed commodity are ...

6. The money income of the consumer and the price of the consumed commodity are fixed and cannot be changed by consumer.



The model:

slope of TU_x (upward)

$$V = U(x) \quad \checkmark$$

$$MU_x = \frac{\partial U}{\partial x} > 0$$

change in consumption is positive
(i.e. satisfaction is increasing)

curvature of TU_x

$$\frac{\partial MU_x}{\partial x} = \frac{\partial^2 U}{\partial x^2} < 0$$

\Rightarrow Concave

1. TU is increasing upwards

2. TU is increasing at a decreasing rate

3. MU is slope of TU

4. TU is concave from second order derivative of TU_x w.r.t. x.

When consumer buys some amount of a commodity, he gains utility but for this gain he must loss some amount of utility which he possess from his own income.

Thus the net utility obtained from a commodity

Thus the net utility obtained from a consumption of commodity x is the utility derived from the commodity - the utility sacrificed for the money loss)

This is denoted by Z , where,

$$Z = \underline{U(x)} - \lambda P_x \cdot x$$

Z = net utility

P_x = price of one unit of commodity.

$U(x)$ = gross utility

x = amount of commodity x consumed.

$\lambda P_x \cdot x$ = money sacrifice for amount of x

λ = converter of money to utility.

$\lambda P_x \cdot x$ = The utility sacrificed for commodity consumption.

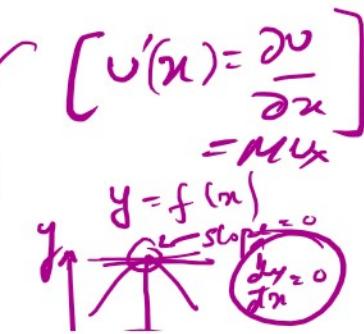
The first order condition of equilibrium commodity consumption of consumer will be

$$Z_{x^*} \frac{\partial Z}{\partial x} = U'(x) - \lambda P_x = 0 \quad \text{or}$$

$$\text{or, } \lambda P_x = U'(x)$$

$$\text{or, } \boxed{\lambda P_x = MU_x} \quad \text{x}$$

∴ second order condition of utility



$$y = f(x)$$

slope = 0

$$\frac{dy}{dx} = 0$$

The second order condition of utility maximisation requires,

$$\frac{\partial^2 Z}{\partial x^2} < 0$$

$$U''(x) < 0$$

$$\boxed{\frac{\partial MU_x}{\partial x} < 0}$$

↳ s.o.c TU_x is concave

↳ i.e. MU_x is diminishing.

\therefore The optimal condition of utility maximisation is $\boxed{MU(x) = \lambda P_x}$

Note: MU is maximised when

$$MU_x = \lambda P_x$$

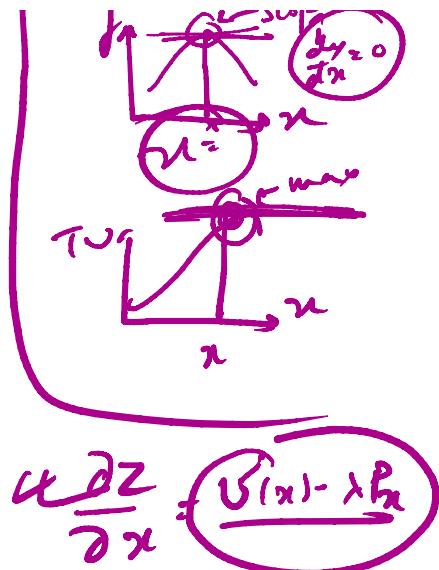
↑ const

$$\text{or, } MU_x \propto P_x \quad \text{--- (1)}$$

According to the law of diminishing MU_x , we know with increase in consumption of x , MU_x is decreasing

$$MU_x \propto \frac{1}{x} \quad \text{--- (2)}$$

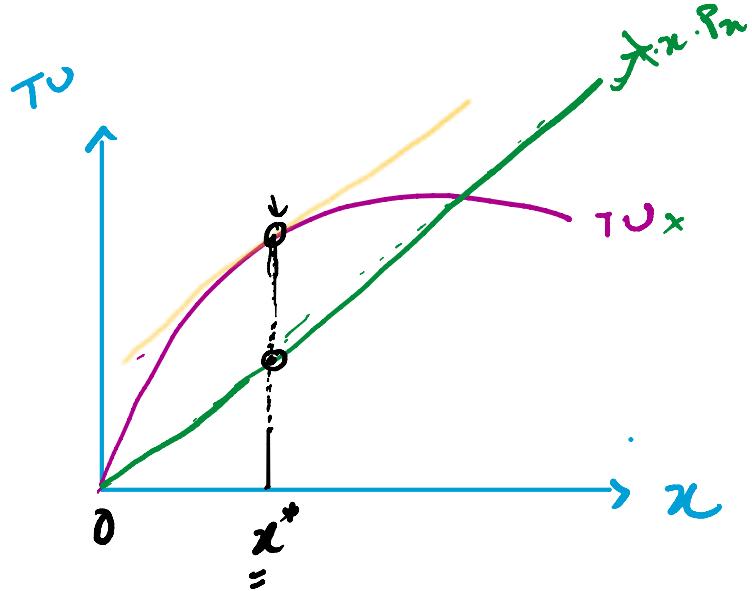
Comparing (1) and (2) $\boxed{P_x \propto \frac{1}{x}}$



Company ① uses $| P_x \propto \frac{1}{x} |$

This shows an inverse relation between price of a commodity 'P_x' and quantity of 'x' consumed.

This inverse relation is the Law of Demand.



Here in the diagram, TU curve goes from origin to the north-east side and is concave to horizontal axis because MU decreases continuously.

Again $\lambda P_x \cdot x$ is a straight line through the origin which crosses the TU curve at a particular point.

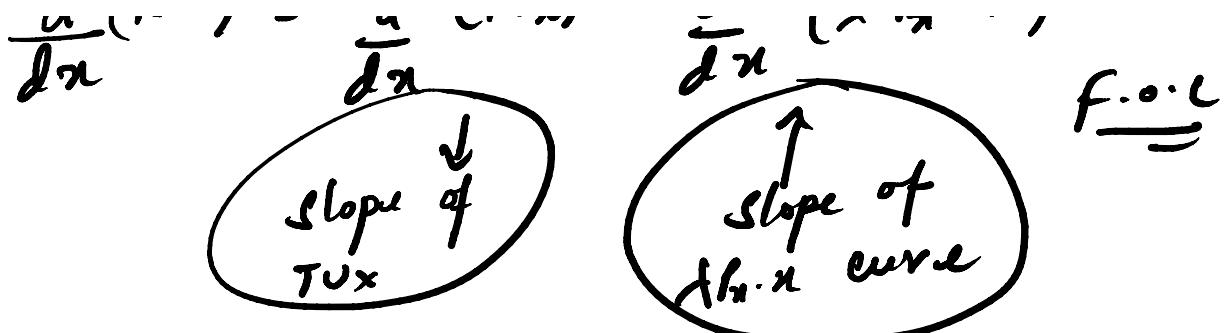
The difference between TU curve and $\lambda P_x \cdot x$ gives net utility and consumer will optimise his consumption at a level where utility is maximised.

$$\text{Thus, } NU = TU_x - \lambda P_x \cdot x$$

x_{NU} is **max** where TU_x is parallel to $\lambda P_x \cdot x$

$$\frac{d}{dx}(NU) = \frac{d}{dx}(TU_x) - \frac{d}{dx}(\lambda P_x \cdot x) = 0$$

F.O.C



$$\text{or, } \frac{d \text{ TU}_x}{d x} = \frac{d}{d x} (x P_x \cdot x)$$

Thus slope of total utility curve = slope of the $x P_x \cdot x$ curve, when consumer is optimized.

Again in terms of marginal utility analysis we have

$$\underline{\underline{MU}_x} = \lambda P_x$$

Second order condition :

$$\frac{d^2 \text{ TU}_x}{d x^2} - \frac{d^2 (x P_x \cdot x)}{d x^2} < 0$$

$$\frac{\partial \text{ MU}_x}{\partial x} - 0 < 0$$

$$\frac{\partial \text{ MU}_x}{\partial x} < 0$$

$\Rightarrow \underline{\underline{MU}_x}$ is downward sloping
and $\underline{\underline{x P_x}}$ is horizontal
(line constant)

Diagramsmetically:

