Q. Mkt demand curve: $Q = 100 - 10P$. There are 10 identical firms in the mkt with the cost fn: $c_i(q_i) = \frac{q_i^2}{2}$, $i = 1, 2, \dots, 10$

(i) Find the pre-tax and post-tax firm and industry output if a per unit tax of 2 is imposed on the producers.

(ii) Find the long-run impact of this per-unit tax.

(i) Mkt demand: $Q = 100 - 10P$.

$n = 10$, $c_i(q_i) = \frac{q_i^2}{2}$, $i = 1, 2, \dots, 10$

Firm's supply curve: π-max condition. $\Rightarrow P = MC_i$

$c_i(q_i) = \frac{q_i^2}{2} \Rightarrow MC_i = q_i$

$\therefore$ Opt. condition: $P = q_i$ or: $\boxed{q_i = P} \Rightarrow$ Firm's supply curve

Mkt supply curve: $Q = \sum_{i=1}^{10} q_i = \sum_{i=1}^{10} P = 10P$.

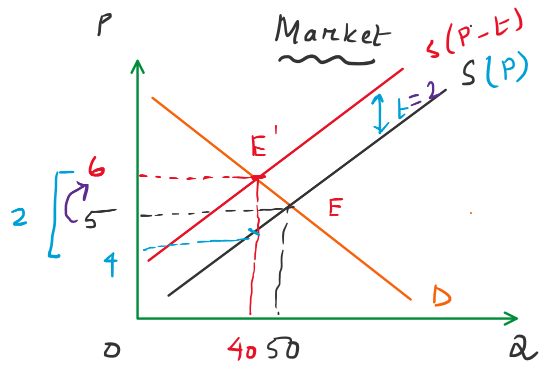
$\therefore$ Mkt dd: $Q = 100 - 10P$ Mkt supply: $Q = 10P$.

Equi: $100 - 10P = 10P$

$100 = 20P \Rightarrow P^* = 5$.

$Q^* = 10 \times 5 = \underline{50}$.

$n = 10$, $q^* = \frac{50}{10} = 5$



$\therefore$ Post-tax: $t = 2$.

Post-tax firm supply: π-max. condition: $P = MC + t$.

Recall: $\pi = P.q - c(q) - t.q$.

$\partial\pi$

Recall: $\pi = P\cdot q - c(q) - t\cdot q$.

$$\frac{\partial \pi}{\partial q} = 0 \Rightarrow P - c'(q) - t = 0 \Rightarrow \boxed{P = c'(q) + t}$$

$MC_i = q_i \quad \therefore$ Opt: $P = MC + t \Rightarrow P = q_i + t$

$$\boxed{q_i = (P-t)} \rightarrow \text{Post-tax firm's supply.}$$

Mkt supply curve: $Q = \sum_{i=1}^{10} q_i = \sum_{i=1}^{10} (P-t) = 10(P-t)$

Mkt supply curve (post-tax): $\boxed{Q = 10(P-2)}$

Post-tax equillibrium: $100 - 10P = 10(P-2)$.

$100 - 10P = 10P - 20$

$120 = 20P \Rightarrow P_d = \frac{120}{20} = 6$.

$\therefore \quad Q^{**} = 40$.

(ii) Long-run impact:

↳ Price will increase by same extent of tax.

↳ All the firms remaining in the mkt will produce pre-tax level of output.

Long Run mkt price $= 5 + 2 = 7$

Long Run firm output $= 5$.

} Find the no. of firms in the mkt.

Long run mkt output $= 100 - 10 \times 7 = 100 - 70 = 30$.

$\therefore$ No. of firms in the long run $= \frac{30}{5} = 6$.

Comparision of Per-unit tax and Ad-valorem-tax

Per unit tax: Govt charges Rs.'t' on every unit of output sold.

If the mkt price $= P$.

then price received by producers (seller's price P_s) = $(P-t)$

$$\pi = R - C = P \cdot q - c(q) - tq$$

$$\pi = (P-t)q - c(q) \quad \text{-------} \quad (*)$$

$$\frac{\partial \pi}{\partial q} = 0 \Rightarrow P - t - c'(q) = 0 \Rightarrow (P-t) = c'(q)$$

$$\text{or} \quad P = c'(q) + t$$

Ad-valorem tax: Govt charges a % say τ. of the price as tax.

Eg: GST : 5% tax on the goods, $\tau = 0.05$

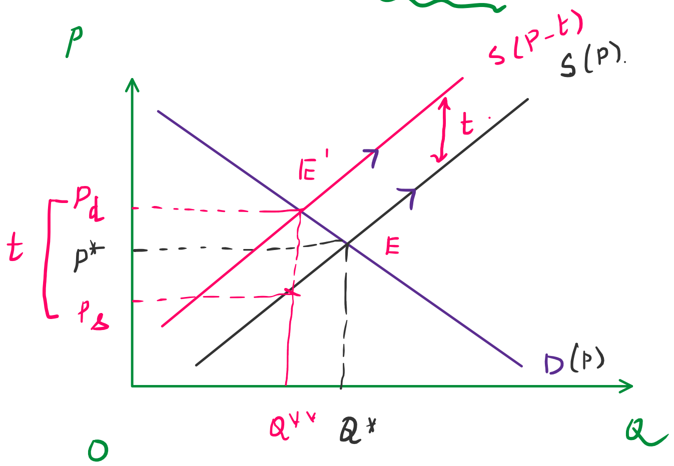
$$\pi = (P - \tau P)q - c(q) \quad \text{---} \quad [\text{ref to eqn } (*)]$$

$$\pi = P(1-\tau) \cdot q - c(q)$$

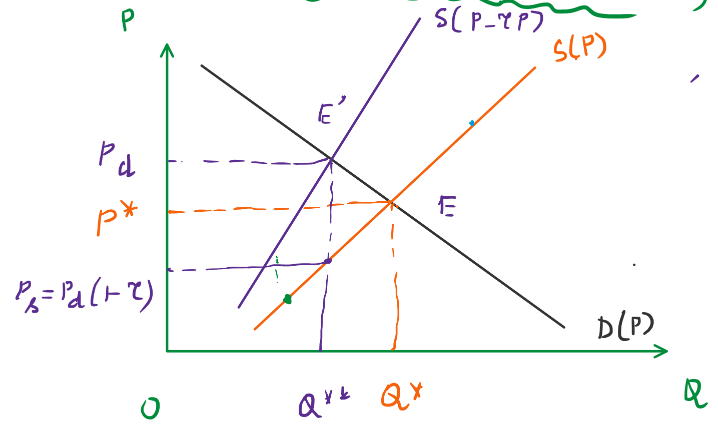
For max: $\frac{d\pi}{dq} = 0 \Rightarrow P(1-\tau) - c'(q) = 0$

$$\Rightarrow P(1-\tau) = c'(q)$$

Mkt (Per-unit tax)



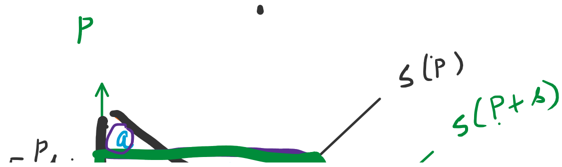
Mkt (Ad-valorem tax)



Note: For a per-unit subsidy of 's' given to the producers by the govt: $\pi = (P+s)q - c(q)$ and then perform the usual π-max.

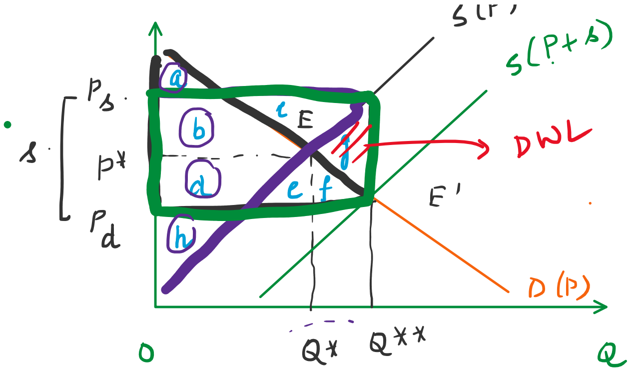
In case of per unit subsidy:

$$\boxed{P_s - P_d = s}$$



$\boxed{P_s - P_d = s}$

In the post-subsidy scenario:-



$CS = a + b + d + e + f$.

$PS = h + b + d + c$.

Cost of subsidy for govt $= -(b + d + c + e + f + g)$

Welfare of society $= (a + \cancel{b} + \cancel{d} + \cancel{e} + \cancel{f}) + (h + b + d + \cancel{c})$
$- (\cancel{b} + \cancel{d} + \cancel{c} + \cancel{e} + \cancel{f} + g) = \underbrace{(a + b + d + h)}_{\text{Pre-subsidy welfare of society}} - g$

Loss of welfare (DWL) $= g$.