OLS

Simple linear regression model

Y (dep)

X (indep)

$$Y = \alpha + \beta X + u$$

u → disturbance term

properties

Sing indep — CLRM

1. $E(u) = 0$
2. $V(u) = \sigma_u^2$ (const) (Homoscedasticity)
3. $E(u_i u_j) = 0$
4. $E(X, u_i) = 0$ → endogeneity

more than one indep — $X_1 X_2 \dots X_n$ — OLS

violation of 2 → heteroskedasticity

3 → autocorrelation

4 → multicollinearity → endogeneity

$$\hat{Y} = \hat{\alpha} + \hat{\beta} X$$

$$Y - \hat{Y} = e$$

OLS: $\hat{\alpha}$ $\hat{\beta}$ estimate such that error is minimised.

$$\sum e^2 = \sum (y-\hat{y})^2 = \sum (y - \hat{\alpha} - \hat{\beta} x)$$

F.O.C

① $\dfrac{\partial \sum e^2}{\partial \hat{\alpha}} = 0$

$\Rightarrow -2\sum (y - \hat{\alpha} - \hat{\beta} x) = 0$

$\Rightarrow \sum y - n\hat{\alpha} - \hat{\beta} \sum x = 0$

$\Rightarrow \sum y = n\hat{\alpha} + \hat{\beta} \sum x$

$\Rightarrow \bar{y} = \hat{\alpha} + \hat{\beta} \bar{x}$

$\Rightarrow \boxed{\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}}$

② $\dfrac{\partial \sum e^2}{\partial \hat{\beta}} = 0$

$\Rightarrow 2\sum (y - \hat{\alpha} - \hat{\beta} x)(-x) = 0$

$\Rightarrow \sum xy - \hat{\alpha}\sum x - \hat{\beta}\sum x^2 = 0$

$$\sum xy = \hat{\alpha} \sum x + \hat{\beta} \sum x^2$$

$\Rightarrow \sum xy = (\bar{y} - \hat{\beta}\bar{x}) \sum x + \hat{\beta} \sum x^2$

$\Rightarrow \sum xy = \bar{y} \sum x - \hat{\beta} \bar{x} \sum x + \hat{\beta} \sum x^2$

$\Rightarrow \sum xy = \bar{y} \sum x + \hat{\beta} (-\bar{x} \sum x + \sum x^2)$

~~$\Rightarrow \sum xy = \bar{y} \sum x + \hat{\beta}(\bar{x}\sum x - \sum x^2) =$~~

$\Rightarrow \hat{\beta}(\sum x^2 - \bar{x}\sum x) = \sum xy - \bar{y}\sum x$

$\Rightarrow \hat{\beta}(\sum x^2 - \bar{x}\sum x) = \sum xy - \bar{y} \sum x$

$\Rightarrow \hat{\beta}\left(\frac{1}{n}\sum x^2 - \bar{x}\frac{\sum x}{n}\right) = \frac{1}{n}\sum xy - \bar{y}\frac{\sum x}{n}$

$\hat{\beta}(\hat{\sigma}_x^2) = \operatorname{cov}(x, y)$

$\hat{\sigma} x$

$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$

$\hat{\beta} = \frac{\Sigma xy - \bar{x}\bar{y}}{\Sigma x^2 - \bar{x}^2}$

$= \frac{\text{cov}(x,y)}{\delta x^2}$

$\Rightarrow \hat{\beta}(\delta_x^2)$

$\Rightarrow \hat{\beta} = \frac{\text{cov}(x,y)}{\delta_x^2}$

MVUE → Min var unbiased estimator

Gauss-Markov theorem → BLUE property

Best, Linear, Unbiased estimator

$\hat{\alpha}$, $\hat{\beta}$ unbiased or not?

$E(\hat{\alpha})$?

$\hat{\beta} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

$= \frac{\Sigma y_i(x_i - \bar{x}) - \bar{y}\Sigma(x_i - \bar{x})}{\Sigma(x_i - \bar{x})^2}$

$= \frac{\Sigma y_i(x_i - \bar{x})}{\Sigma(x_i - \bar{x})^2}$

$\hat{\beta} = \Sigma k_i y_i$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{X}$

$\hat{\alpha} = \frac{1}{n}\Sigma y_i - \bar{X}(\Sigma k_i y_i)$

$\hat{\alpha} = \Sigma\left[\frac{1}{n} - \bar{X}k_i\right] y_i$

$\bar{x}k_i] (\alpha + \beta x_i)$

$\hat{\beta} = \sum k_i Y_i = \frac{\sum (Y_i ...)}{\sum (x_i - \bar{x})}$

let $x_i = (X_i - \bar{X})$, $\hat{\beta} = \frac{\sum Y_i x_i}{\sum x_i^2}$

$\hat{\alpha} = \sum \left[\frac{1}{n} - \bar{X} k_i \right] (\alpha + \beta X_i + u_i)$

$\hat{\alpha} = \frac{1}{n}\sum \alpha + \frac{\beta}{n} \sum X_i - \bar{X}\alpha \sum k_i - \beta \bar{X} \sum k_i X_i$
$+ \frac{1}{n} \sum u_i - \bar{X} \sum k_i u_i$

$x_i = X_i - \bar{X}$
$X_i = x_i + \bar{X}$

$\hat{\alpha} = \frac{1}{n} \sum \alpha + \beta \bar{X} - \beta \bar{X} + \frac{1}{n} \sum u_i - \bar{X} \sum k_i u_i$

$k_i = \frac{x_i}{\sum x_i^2}$

$\sum k_i = \frac{\sum x_i}{\sum x_i^2} = 0$ ✓

$\sum k_i X_i = \sum k_i (x_i + \bar{X})$
$= \sum k_i x_i + \bar{X} \sum k_i$
$= \sum k_i x_i$
$= \frac{\sum x_i \cdot x_i}{\sum x_i^2} = \frac{\sum x_i^2}{\sum x_i^2} = 1$

$\hat{\alpha} = \frac{1}{n} \sum \alpha + \frac{1}{n} \sum u_i - \bar{X} \sum k_i u_i$

$E(\hat{\alpha}) = E(\alpha) + \frac{1}{n} \sum E(u_i) - \bar{X} \sum k_i E(u_i)$

$E(\hat{\alpha}) = \alpha$ $\quad E(\hat{\alpha}) - \alpha = 0$

$\hat{\alpha} \to$ stat
$\alpha \to$ param

$\therefore E(\hat{\alpha}) = \alpha$ $\quad \therefore \hat{\alpha}$ is unbiased estimator α.

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Multiple Regression Model

$... \beta X_{1i} + \beta X_{2i} + u_i$

Multiple reg

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$$

$$e_i = Y_i - \hat{Y}$$

$$\Sigma e_i^2 = \Sigma (Y_i - \hat{Y})^2$$

$$= \Sigma (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i})^2$$

$$\frac{\partial \Sigma e_i^2}{\partial \hat{\beta}_1} = 0 \Rightarrow -2\Sigma (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i}) = 0$$

$$\Rightarrow \Sigma (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i}) = 0$$

$$\Sigma Y_i = n\hat{\beta}_1 + \hat{\beta}_2 \Sigma X_{2i} + \hat{\beta}_3 \Sigma X_{3i}$$

or $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}_2 - \hat{\beta}_3 \bar{X}_3$ — (1)

$$\frac{\partial \Sigma e_i^2}{\partial \hat{\beta}_2} = 0$$

$$\Rightarrow -2\Sigma X_{2i} (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i}) = 0$$

$$\Sigma X_{2i} Y_i = \hat{\beta}_1 \Sigma X_{2i} + \hat{\beta}_2 \Sigma X_{2i}^2 + \hat{\beta}_3 \Sigma X_{2i} X_{3i}$$

or

$$\Sigma X_{2i} Y_i = (\bar{Y} - \hat{\beta}_2 \bar{X}_2 - \hat{\beta}_3 \bar{X}_3) \Sigma X_{2i} + \hat{\beta}_3 \Sigma X_{2i} X_{3i}$$

$$+ \hat{\beta}_2 \Sigma X_{2i}^2$$

$$\Sigma X_{2i})$$

Σx_{2i} ... $+ \hat{\beta}_2 \Sigma x_{2i}$ ...)

or, $\Sigma X_{2i} Y_i - \bar{Y} \Sigma X_{2i} = \hat{\beta}_2 \left(\Sigma X_{2i}^2 - \bar{X}_2 \Sigma X_{2i} \right) + \hat{\beta}_3 \left(\Sigma X_{2i} X_{3i} - \bar{X}_3 \Sigma X_{2i} \right)$

$\Sigma x_2 y$; Σx_2^2 ; Σx_3^2

Result: $\Sigma X_i Y_i - \bar{Y} \Sigma X_i = \Sigma (X_i - \bar{X})(Y_i - \bar{Y}) = \Sigma x_i y_i$

$\Sigma X_i^2 - \bar{X} \Sigma X_i = \Sigma (X_i - \bar{X})^2 = \Sigma x_i^2$

$$\Sigma x_2 y = \hat{\beta}_2 \Sigma x_2^2 + \hat{\beta}_3 \Sigma x_2 x_3 \quad \text{--- (3)}$$

(3) $\dfrac{\partial \Sigma e_i^2}{\partial \hat{\beta}_3} = 0$