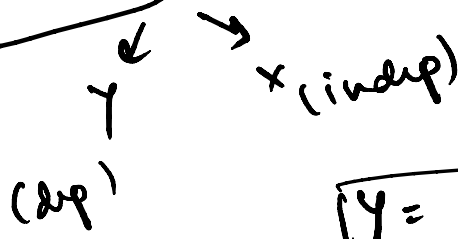


Simple linear regression model

OLS



$$Y = \alpha + \beta X + u$$

disturbance term properties

simy indep CLRM

more than one indep X_1, X_2, \dots, X_n

OLS

- 1 $E(u) = 0$
- 2 $V(u) = \sigma_u^2$ (const) (Homoscedasticity)
- 3 $E(u_i u_j) = 0$
- 4 $E(X_i u_i) = 0$

endogeneity

violation of 2 →

heteroskedasticity
auto correlation

3 →

multi collinearity
endogeneity

$$\hat{Y} = \hat{\alpha} + \hat{\beta} X$$

OLS

$$Y - \hat{Y} = e$$

$\hat{\alpha}$ $\hat{\beta}$ estimate such that error is minimized.

$$\sum e^2 = \sum (y - \hat{y})^2 = \sum (y - \hat{\alpha} - \hat{\beta}x)$$

F.O.C

$$\textcircled{1} \frac{\partial \sum e^2}{\partial \hat{\alpha}} = 0$$

$$\Rightarrow -2 \sum (y - \hat{\alpha} - \hat{\beta}x) = 0$$

$$\Rightarrow \sum y - n\hat{\alpha} - \hat{\beta} \sum x = 0$$

$$\Rightarrow \sum y = n\hat{\alpha} + \hat{\beta} \sum x$$

$$\Rightarrow \bar{y} = \hat{\alpha} + \hat{\beta} \bar{x}$$

$$\Rightarrow \hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

$$\textcircled{2} \frac{\partial \sum e^2}{\partial \hat{\beta}} = 0$$

$$\Rightarrow 2 \sum (y - \hat{\alpha} - \hat{\beta}x) \cdot (-x) = 0$$

$$\Rightarrow -2 \sum (y - \hat{\alpha} - \hat{\beta}x)x = 0$$

$$\Rightarrow \sum (y - \hat{\alpha} - \hat{\beta}x)x = 0$$

$$\Rightarrow \sum xy = \hat{\alpha} \sum x + \hat{\beta} \sum x^2$$

$$\Rightarrow \sum xy = (\bar{y} - \hat{\beta} \bar{x}) \sum x + \hat{\beta} \sum x^2$$

$$\Rightarrow \sum xy = \bar{y} \sum x - \hat{\beta} \bar{x} \sum x + \hat{\beta} \sum x^2$$

$$\Rightarrow \sum xy = \bar{y} \sum x + \hat{\beta} (\sum x^2 - \bar{x} \sum x)$$

$$\hat{\beta} (\sum x^2 - \bar{x} \sum x) = \sum xy - \bar{y} \sum x$$

$$\hat{\beta} \left(\frac{1}{n} \sum x^2 - \bar{x} \frac{\sum x}{n} \right) = \frac{1}{n} \sum xy - \bar{y} \frac{\sum x}{n}$$

$$\hat{\beta} \left(\frac{\sum x^2}{n} - \bar{x} \frac{\sum x}{n} \right) = \frac{\sum xy - \bar{y} \sum x}{n}$$

$$\hat{\beta} \left(\frac{\sum (x - \bar{x})^2}{n} \right) = \frac{\sum (x - \bar{x})(y - \bar{y})}{n}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

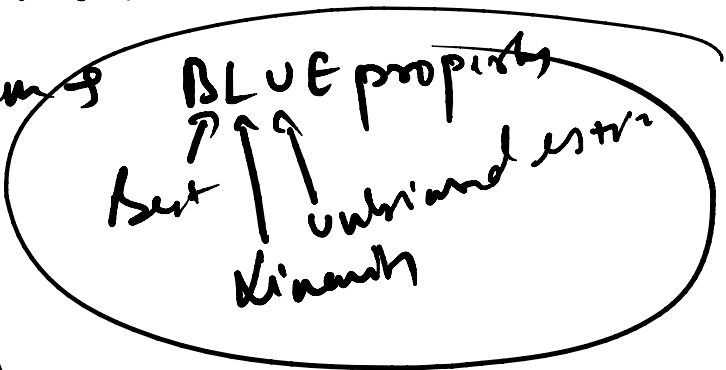
$$\hat{\beta} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \frac{\text{Cov}(x, y)}{\sigma_x^2}$$

$$\Rightarrow \hat{\beta} = \frac{\text{Cov}(x, y)}{\sigma_x^2}$$

MVUE → Min var unbiased estimator

Gauss-Markov theorem



$$E(\hat{\alpha}) = ?$$

unbiased or not?

$$\hat{\beta} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \frac{\sum y_i (x_i - \bar{x}) - \bar{y} \sum (x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

$$\sum y_i = \sum k_i y_i = \sum y_i (x_i - \bar{x}) / (x_i - \bar{x})$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

$$\hat{\alpha} = \frac{1}{n} \sum y_i - \bar{x} \left(\frac{\sum k_i y_i}{\sum (x_i - \bar{x})^2} \right)$$

$$\hat{\alpha} = \sum \left[\frac{1}{n} - \frac{\bar{x} k_i}{\sum (x_i - \bar{x})^2} \right] y_i$$

$$\hat{\beta} = \frac{\sum (x_i - \bar{x}) y_i}{\sum (x_i - \bar{x})^2}$$

$$\hat{\alpha} = \frac{1}{n} \sum y_i - \bar{x} \hat{\beta}$$

$$\hat{\alpha} = \frac{1}{n} \sum y_i - \bar{x} \left[\frac{\sum (x_i - \bar{x}) y_i}{\sum (x_i - \bar{x})^2} \right]$$

$$\hat{\alpha} = \frac{1}{n} \sum y_i + \beta \bar{x} - \beta \bar{x} \frac{\sum (x_i - \bar{x}) y_i}{\sum (x_i - \bar{x})^2}$$

$$K_i = \frac{x_i - \bar{x}}{\sum (x_i - \bar{x})^2}$$

$$\sum K_i = 0$$

$$\sum K_i x_i = \sum K_i (x_i - \bar{x}) + \bar{x} \sum K_i$$

$$= \sum K_i x_i - \bar{x} \sum K_i + \bar{x} \sum K_i$$

$$\hat{\alpha} = \frac{1}{n} \sum y_i + \frac{1}{n} \sum u_i - \bar{x} \sum K_i u_i$$

$$= \frac{\sum y_i}{n} + \frac{\sum u_i}{n} - \frac{\sum x_i u_i}{\sum (x_i - \bar{x})^2}$$

$$E(\hat{\alpha}) = E\left(\frac{1}{n} \sum y_i\right) + \frac{1}{n} \sum E(u_i) - \bar{x} \sum K_i E(u_i)$$

$$E(\hat{\alpha}) = \alpha + \frac{1}{n} \sum E(u_i) - \bar{x} \sum K_i E(u_i)$$

$\hat{\alpha} \rightarrow \text{stat}$
 $\alpha \rightarrow \text{param}$

$$\therefore E(\hat{\alpha}) = \alpha$$

$\therefore \hat{\alpha}$ is unbiased estimator α .

— * —

Multiple Regression Model

$$y_i = \alpha + \beta x_i + u_i$$

multiple

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i}$$

$$e_i = Y_i - \hat{Y}_i$$

$$\sum e_i^2 = \sum (Y_i - \hat{Y}_i)^2$$

$$= \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i})^2$$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_1} = 0$$

$$\Rightarrow -2 \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i}) = 0$$

$$\Rightarrow \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i}) = 0$$

$$\sum Y_i = n \hat{\beta}_1 + \hat{\beta}_2 \sum X_{2i} + \hat{\beta}_3 \sum X_{3i}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}_2 - \hat{\beta}_3 \bar{X}_3 \quad (1)$$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}_2} = 0$$

$$\Rightarrow -2 \sum X_{2i} (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_{2i} - \hat{\beta}_3 X_{3i}) = 0$$

$$\sum X_{2i} Y_i = \hat{\beta}_1 \sum X_{2i} + \hat{\beta}_2 \sum X_{2i}^2 + \hat{\beta}_3 \sum X_{2i} X_{3i}$$

or

$$\sum X_{2i} Y_i = \left(\bar{Y} - \hat{\beta}_2 \bar{X}_2 - \hat{\beta}_3 \bar{X}_3 \right) \sum X_{2i} + \hat{\beta}_2 \sum X_{2i}^2 + \hat{\beta}_3 \sum X_{2i} X_{3i}$$

$$\sigma_1 \sum x_{2i} y_i - \bar{y} \sum x_{2i} = \hat{\beta}_2 \sum x_{2i} + \hat{\beta}_3 \sum x_{2i} x_{3i} - \bar{y} \sum x_{2i}$$

$$= \hat{\beta}_2 (\sum x_{2i} - \bar{x}_2 \sum x_{2i}) + \hat{\beta}_3 \sum x_{2i} x_{3i}$$

Result: $\sum x_{2i} y_i - \bar{y} \sum x_{2i} = \sum (x_{2i} - \bar{x}_2) (y_i - \bar{y}) + \sum x_{2i} y_i$

$$\sum x_{2i}^2 - \bar{x}_2 \sum x_{2i} = \sum (x_{2i} - \bar{x}_2)^2 = \sum x_{2i}^2$$

$$\sum x_{2i} y_i = \hat{\beta}_2 \sum x_{2i}^2 + \hat{\beta}_3 \sum x_{2i} x_{3i} \quad \text{--- (3)}$$

(3) $\frac{\partial \sum e_i^2}{\partial \hat{\beta}_3} = 0$