20 December 2023 17:31

Number System

① Find the last digit in powers of numbers

find the last digit in the expansion of 2^{2003}.

last digit									
	1	2	3	4	5	6	7	8	9
Power = 2	1	4	9	6	5	6	9	4	1
Power = 3	1	8	7	4	5	6	3	2	9
Power = 4	1	6	1	6	5	6	1	6	1
Power = 5	1	2	3	4	5	6	7	8	9

→ 2, 3, 7, 8

→ 2, 3, 7, 8, 4, 9

$512 \times 8 = \_6$

$729 \times 9 = \_1$

$1^2 = 1$

$4^2 = 16$

$5^2 = 25$

$3^3 = 27$

$7^4 = 49 \times 49 = 343 \times 7 = 2401$

No perfect square can end in 2, 3, 7, 8

$2^{8} = 256$ $\text{Rem}\left(\frac{8}{4}\right) = 0$, $\therefore 2^8 \Rightarrow 2^4 \Rightarrow 6$

2^{2003} $\text{Rem}\left(\frac{2003}{4}\right) = 3$ $2^{2003} \Rightarrow 2^3 \Rightarrow 8$

$4 \rightarrow 4, 6.$ $9 \rightarrow 9, 1$

$4^{odd} \Rightarrow 4$, $4^{even} \Rightarrow 6$ (10−4) $9^{odd} \Rightarrow 9$, $9^{even} \Rightarrow 1$ (10−9)

10

$2 \rightarrow 2, 4, 8, 6.$ (① ② ③ ④; 10, 10, 10)

$2^1 = 2$

$2^2 = 4$

$\therefore 2^3 \rightarrow 10 - 2 = 8$

$\therefore 2^4 \rightarrow 10 - 4 = 6$

$8 \rightarrow 8, 4, 2, 6.$ (10, 10)

$8^1 = 8$

$8^2 = 4$

$\therefore 8^3 \rightarrow 10 - 8 = 2$

$8^4 \rightarrow 10 - 4 = 6$

$3 \rightarrow 3, 9, 7, 1$

$3^1 = 3$

$3^2 = 9$

$\therefore 3^3 \rightarrow 10 - 3 = 7$

$\therefore 3^4 \rightarrow 10 - 9 = 1$

$3 \rightarrow 3, 9, 7, 1$

$\boxed{3^1 = 3 \quad 3^2 = 9}$

$\therefore 3^3 \rightarrow 10-3 = 7.$

$\therefore 3^4 \rightarrow 10-9 = 1$

10-7 10-9

$7 \rightarrow 7, 9, 3, 1$

3^{379}

$R\left(\frac{379}{4}\right) \Rightarrow R\left(\frac{79}{4}\right) = 3.$

$3^1 = 3$

$\Downarrow$

$3^3 \Rightarrow 10-3 = 7$

$$\frac{\cancel{27}3^{372} \times \cancel{37}2^{273}}{} \Rightarrow 3^{372} \times 2^{273}$$

$\Downarrow$ 2

$3^4 \quad 2^1$

$1 \times 2 = 2$

2, 3, 7, 8 → divide the power by and take the remainder. (if remainder = 0 then power = 4)

4, 9 → Check whether the power is odd or even

② Finding the Remainder.

find the remainder when $\frac{2^{203}}{9}$.

what can be the remainders when any no is divided by 9. → $\boxed{0 \text{ to } 8}$

$N = 9 \times Q + R.$

$6 = 9 \times 0 + 6.$

$= \boxed{9 \times 1 + (-3)}$ negative remainder.

$2^{203} = 2^{201+2} = 2^{201} \times 2^2 = 2^{3 \times 67} \times 2^2.$

$2^{203} = (2^3)^{67} \times 2^2.$

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$R\left(\frac{2^{203}}{9}\right) = R\left[\frac{(2^3)^{67}}{9}\right] \times R\left[\frac{2^2}{9}\right]$

$2^3 = 8$

$= \left[R\left(\frac{2^3}{9}\right)\right]^{67} \times 4$

$8 = 9\times0+8$

$= 9\times1+(-1)$

$= (-1)^{67} \times 4 = (-1)\times4 = -4$

$5 = 9\times0+5$

$= 9\times1+(-4)$

Actual rem $= 9+(-4) = 5$

Target → ① express the remainder as ± 1.

② " " " " ± 2

$\text{Rem}\left(\frac{7^{36}}{9}\right)$

$\boxed{\text{Rem}\left(\frac{7}{9}\right) = -2}$ $\boxed{\text{Rem}\left(\frac{343}{9}\right) = 1}$

$\text{Rem}\left(\frac{7^2}{9}\right) = 4$

$R\left(\frac{7^{3\times12}}{9}\right)$

$R\left[\frac{(7^3)^{12}}{9}\right] \Rightarrow \left[R\left(\frac{7^3}{9}\right)\right]^{12} \Rightarrow 1^{12} = 1$

$R\left(\frac{7^{36}}{9}\right) \Rightarrow \left[R\left(\frac{7}{9}\right)\right]^{36} \Rightarrow (-2)^{36}$.

$-2^{36} = 2^{36}$,

$R\left(\frac{2^{36}}{9}\right) \Rightarrow R\left(\frac{2^{3\times12}}{9}\right) \Rightarrow R\left[\frac{(2^3)^{12}}{9}\right]$

$R\left(\frac{8}{9}\right) = -1$

$\Rightarrow \left[R\left(\frac{2^3}{9}\right)\right]^{12} \Rightarrow (-1)^{12} = 1$

Find the smallest number which when divided by 3, 4 and 5 ... of 1 in each case

Find the smallest number which when divided by 3, 4 and 5 gives a remainder of 1 in each case.

$$LCM(3,4,5) = 60.$$
$$+1 = 61$$

Find the smallest no which when divided by 3 gives a rem of 2, when divided by 4 gives a rem of 3 and when divided by 5 gives a rem of 4.

$$R\left(\frac{N}{3}\right) = 2,\ 2-3 \Rightarrow 2, -1$$
$$R\left(\frac{N}{4}\right) = 3,\ 3-4 \Rightarrow 3, -1$$
$$R\left(\frac{N}{5}\right) = 4,\ 4-5 \Rightarrow 4, -1$$

$$LCM(3,4,5) = 60.$$
$$+(-1) = 59$$

Find the remainder when $\left(\frac{2^{12}}{7}\right) \Rightarrow$

$$2^{12} = 2^{3\times4} = (2^3)^4. \qquad R\left(\frac{2^3}{7}\right) = 1$$
$$1^4$$
$$1$$

$$2^{12} = 2^{6\times2} = (2^6)^2 \qquad R\left(\frac{2^{6}}{7}\right) \quad 6 = 7-1$$
p.

Fermat's Rule $R\left(\frac{n^{p-1}}{p}\right) = 1$ where p is a prime no > 3

$$R\left(\frac{73^{24}}{13}\right) \qquad 24 = 12\times2$$
$$\frac{73^{12}}{13} \rightarrow 1$$

$$R\left(\frac{3^{52}}{17}\right) \qquad 17-1 = 16$$
$$52 = 16\times3 + 4$$

$3^{52} = 3^{16\times 3} \times 3^4 = (3^{16})^3 \times 3^4.$ $\quad R\left(\frac{3^{16}}{17}\right) = 1$

$\Rightarrow 1^3 \times R\left(\frac{3^4}{17}\right) \Rightarrow R\left(\frac{81}{17}\right) \Rightarrow 13.$