23 July 2023 17:17

Real Nos. → any no on the number line

$\boxed{R^2 \geq 0}$

-2 -1 0 1 2

$(-2)^2 = 4$ $0^2 = 0$. $(2)^2$ 4

$(a-2)^2 + (b-3)^2 + (c-4)^2 = 0$ a, b, c = ?

$(a-2)^2 = 0$ $(b-3)^2 = 0$ $(c-4)^2 = 0$. a = 2, b = 3, c = 4

$\boxed{AM \geq GM} \geq HM$.

$AM(a,b) = \frac{a+b}{2}$.

$GM(a,b) = \sqrt{ab}$.

$HM(a,b) = \frac{2ab}{a+b}$.

a, b are 2 real nos.

$\therefore$ $(a-b)$ is a real no.

$\therefore$ $(a-b)^2 \geq 0$.

$a^2 - 2ab + b^2 \geq 0$.

$a^2 + b^2 \geq 2ab$.

$\frac{a^2+b^2}{2} \geq ab$.

$\boxed{AM \geq GM}$.

$AM(a^2, b^2) = \frac{a^2+b^2}{2}$

$GM(a^2, b^2) = \sqrt{a^2b^2} = ab$.

$a^2 \geq 0$.

$b^2 \geq 0$.

applicable for non-negative real numbers

$\therefore$ sum of a^2 and $b^2 \geq$ product of 2ab.

If a and b are 2 non-negative real nos then $a+b \geq 2\sqrt{ab}$.

a+b = 10 find the maximum value of ab. where a, b are positive integers.

a	b	a×b
1	9	9
2	8	16.
3	7	21
4	6	24
5 ×	5 =	25.
6	4	24.
7	3	21
8	2	16.
9	1	9

max. a = b = 5

a+b = 16. $(ab)_{max} = 8 \times 8 = 64$

a = b = 8

a+b = 18 $(ab)_{max} = 81$

a+b = 14 $(ab)_{max} = 49$

a+b+c = 18 $(abc)_{max} = 216$

a = b = c = 6

LHS RHS

AM ≥ GM

$a^2+b^2 \geq 2ab$.

$a^2+b^2 \geq 2ab$.

$a^2+b^2 \geq 2ab.$ $\qquad\qquad$ $a^2+b^2 \geq \underline{2ab}.$

$\geq(\ )$ $\qquad\qquad$ $(\ )\geq$

$(a^2+b^2)_{min} = 2ab.$ $\qquad\qquad$ $a^2+b^2 = (2ab)_{max}$

$$(a^2+b^2)_{min} = (2ab)_{max}$$

$$a^2+b^2 = 2ab,$$

$$a^2-2ab+b^2=0.$$

$$(a-b)^2=0. \quad \boxed{a=b}$$

Imp. If the sum of numbers is given then their product will be maximum when the numbers are equal.

If the product of numbers is given then their sum will be minimum when the numbers are equal.

$\underline{ab=16}.$ find the minimum value of $(a+b)$ where a, b are positive integers.

$a=b=\sqrt{ab}$

$\boxed{b=\frac{16}{a}.}$

a	b	$a+b$
1 +	16	17
2 +	8	10
4 ·	4	8
8	2.	10.
16	1	17

min when $a=b=4$.

$ab=36$

$a=b=6$ $\qquad$ $(a+b)_{min} = a+b = \underline{12}$

$ab=49$ $\qquad$ $(a+b)_{min} = 14$

$ab=196.$ $\qquad$ $(a+b)_{min} = 28$

<u>If the sum of 2 numbers is given.</u>

$a+b=c.$ $\qquad$ $a=b=c/2.$ $\qquad$ $(ab)_{max} = \left(\frac{c}{2}\right)^2.$

$a+b=10$ $\qquad$ $a=b=10/2 = 5$ $\qquad$ $(ab)_{max} = (5)^2 = 25.$

<u>If the sum of more than 2 numbers is given</u>

If the sum of more than 2 numbers is given

$a+b+c+d = 32$. $a=b=c=d = \frac{32}{4} = 8$

$(abcd)_{max} = 8\times8\times8\times8$

$= 8^4 = 2^{12} = 4096$

$abcd = 81$ $\overset{3+3+3+3}{(a+b+c+d)}_{min} = 12$

$a.a.a.a = 81$

$a^4 = 81$

$a = (81)^{1/4}$

$a=b=c=d=(81)^{1/4}$

$81 = 3^4$

$= (3^4)^{1/4} = 3^{4\times\frac{1}{4}} = 3$

$abcd = 256$ $(a+b+c+d)_{min} = 16$.

$a=b=c=d=(256)^{\frac{1}{4}}$

$256 = 2^8$

$(2^8)^{1/4} = 2^{8\times\frac{1}{4}} = 4$ $= 4$

If x is a real no such that $x > 0$

find the minimum value of $x + \frac{1}{x}$.

$a+b \geq 2\sqrt{ab}$.

$x + \frac{1}{x} \geq 2\sqrt{x\times\frac{1}{x}}$

$x \times \frac{1}{x} = 1$

$\sqrt{x\times\frac{1}{x}} = 1$

$x + \frac{1}{x} \geq 2$.

$(x+\frac{1}{x})_{min} = 2$.

Example: Let a, b and c be nonnegative integers such that a + b+ c = 15. What is the maximum value of a.b.c + a.b + b.c + c.a?

$(abc+ab)+(bc+ca)$

$= ab(c+1) + c(b+a)$

$abc + bc + ab + ca$

$bc(a+1)$

$(a+1)+(b+1)+(c+1) \geq 2\sqrt{(a+1)(b+1)(c+1)}$

$a+b+c+3 \geq 2\sqrt{abc+ab+bc+ac+16}$

$18 \geq 2\sqrt{abc+ab+bc+ac+16}$

$9 \geq \sqrt{abc+ab+bc+ac+16}$.

$81 \geq abc+ab+bc+ac+16$.

$(a+1)(b+1)$

$= ab+a+b+1$

$(a+1)(b+1)(c+1)$

$= (ab+a+b+1)(c+1)$

$= abc+ab+ac+a+bc+b+c+1$

$= (abc+ab+bc+ac)+a+b+c+1$

$$81 \geq abc + ab + bc + ac + 16.$$

$$65 \geq abc + ab + bc + a$$

$$= abc + ab + ac + a + bc + b + c + 1$$

$$= (abc + ab + bc + ac) + \underline{\underline{a+b+c+1}}$$

$$= (abc + ab + bc + ac) + 16.$$

$$abcd = 625.$$

$$a = b = c = d = (625)^{1/4},$$

$$625 = 5^4$$

$$(5^4)^{1/4} = 5^{4 \times \frac{1}{4}} = 5$$

$$(a^b)^c = a^{bc}.$$

$$a^b \cdot a^c = a^{b+c}$$

$$\frac{a^b}{a^c} = a^{b-c}.$$