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$\frac{\Delta Y}{\Delta K} = \sigma$

$\Delta Y = \sigma \Delta K$

$\Delta K = I$ ← change in stock of capital

Investment multiplier,

$\frac{\Delta Y}{\Delta I} = \frac{1}{1-mpc}$

$\frac{\Delta Y}{\Delta I} = \frac{1}{mps}$

$\alpha = mps$

$\frac{\Delta Y}{\Delta I} = \frac{1}{\alpha}$

$\Delta Y = \frac{\Delta I}{\alpha}$

The Domar Model:

A Russian American economist, Evsey David Domar (April 16, 1914 – April 1, 1997), builds his model from both demand as well as the supply side based on dual effect of investment and provided the solution for steady growth.

To simplify the model, the demand and the supply equation in the incremental form can be written as follows:

The demand side of the long-term effect of investment can be summarized and expressed through the following relation as:

demand

$\Delta Y_d = \Delta I\ (1/\alpha)$ [Change in income (ΔY_d) equals multiplier ($1/\alpha$) times the Change in investment (ΔI)]

Or $\Delta Yd = \frac{\Delta I}{\alpha}$ ……………(1)

Where;

α (Alpha) = mps = Marginal propensity to save which is reciprocal of multiplier.

The supply size of investment can be summarized and expressed through the following relation as:

supply

$\Delta Y_s = \sigma \Delta K$ [Change in output supply (ΔY_s) equals the product of Change in real capital (ΔK) and capital Productivity (σ)]

Or $\Delta Y_s = \sigma I$ ………..(2) [Since ΔK=I where I denotes Net investment]

equilibrium: $\Delta Y_d = \Delta Y_s$

$\frac{\Delta I}{\alpha} = \sigma I$ or, $\boxed{\frac{\Delta I}{I} = \sigma \cdot \alpha}$ ← equilibrium (Domar's model) or $\frac{\Delta Y}{Y}$

Domar's condition of steady state growth can be explained with the help of numerical example.

Suppose the productivity of capital (σ) is 25% and the marginal propensity to save ((α) is12%, then;

$\sigma = 25\% = 25/100$

$mps = \alpha = 12\% = 12/100$

$\frac{\Delta Y}{Y} = \frac{25}{100} \times \frac{12}{100} = \frac{3}{100}$ or 3%

Or $\frac{\Delta I}{I} = \frac{25}{100} \times \frac{12}{100} = \frac{3}{100}$ or 3%

$\frac{\Delta Y}{Y} = \frac{\Delta I}{I} = \sigma\alpha$

$\frac{\Delta Y}{Y} \neq \frac{\Delta I}{I} \neq \sigma\alpha$

Thus, the above numerical example shows that income and investment must grow at an annual rate of 3% if steady growth rate is to be maintained at full-employment. Any divergence from this 'golden path' will lead to cyclical fluctuations. Disequilibrium reflecting non-steady growth state would prevail if:

1) $\frac{\Delta I}{I} > \alpha\sigma$ and the economy would experience inflation.
2) $\frac{\Delta I}{I} < \alpha\sigma$ and the economy would suffer from secular stagnation.

(Disequilibrium)

$\frac{\Delta I}{I} > \sigma\alpha$ excess demand inflation

$\frac{\Delta I}{I} < \sigma\alpha$ ⇒ excess supply ⇒ stagnation

Statistics → Normal Distribution

↳ continuous random variable (x) → pdf

(x) to be pdf

1. $f(x) > 0\ \forall x$

and

2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{\infty} f(x)\,dx$

pdf ✓

pdf of x ~ Normal distribution with parameter mean (μ) and variance (σ^2) is given by [i.e $\boxed{x \sim N(\mu, \sigma^2)}$] ✓

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad ; \quad (-\infty < x < \infty)$$

Property 1: we have to check $\int_{-\infty}^{\infty} f(x)\,dx = 1$ or not.

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\,dx$$

[let $t = \frac{x-\mu}{\sigma}$

then, $\frac{dt}{dx} = \frac{1}{\sigma}$

or, $dx = \sigma\,dt$]

$$= \int_{-\infty}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-t^2/2}\,\cancel{\sigma}\,dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2}\,dt.$$

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t^2/2}\,dt$$

[$\because e^{-t^2/2}$ is even fn]

$e^{-t^2/2}\,dt$ even integ fn.

let $y = t^2/2 \Rightarrow t^2 = 2y$, $t = \sqrt{2y}$

$\frac{dy}{dt} = \cancel{2}t/\cancel{2}$

$\boxed{dt = \frac{dy}{t}}$

$dt = dy/\sqrt{2y}$

$$= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} e^{-y} \frac{dy}{\sqrt{2y}}$$

$$= \frac{\cancel{2}}{2\sqrt{\pi}} \int_0^{\infty} e^{-y} y^{-1/2}\,dy$$

$\Gamma(1/2)$

$$= \frac{1}{\sqrt{\pi}} \times \Gamma(1/2)$$

[$\because \int_0^{\infty} e^{-y} y^{-1/2}\,dy = \Gamma(1/2) = \sqrt{\pi}$]

$$= \frac{1}{\cancel{\sqrt{\pi}}} \times \cancel{\sqrt{\pi}}$$

$= 1$ (ans)

Property 2: Mean of Normal distribution is $E(x) = \mu$.

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} x \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

[Let, $t = \frac{x-\mu}{\sigma}$

then $\boxed{x = \sigma t + \mu}$

$dt/dx = 1/\sigma$

$dx = \sigma \cdot dt$]

$$E(x) = \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} \int_{-\infty}^{\infty} (\sigma t + \mu)\, e^{-\frac{1}{2}t^2}\, \cancel{\sigma}\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma t + \mu)\, e^{-1/2\, t^2}\, dt$$

$$= \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2\, t^2}\, dt + \underbrace{\frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t \cdot e^{-1/2\, t^2}\, dt}_{= 0}$$

[$\because$ $t \cdot e^{-1/2 t^2}$ is even fn $\therefore \int t \cdot e^{-1/2 t^2} dt = 0$ | $e^{-t^2/2}$ is even fn $\therefore \int_{-\infty} e^{-t^2/2} \neq 0$]

$\left[\therefore \int t \cdot e^{-\frac{1}{2}t^2} dt = 0 \quad \middle| \quad \therefore \int_{-\infty} e^{-t^2/2} \neq 0 \right]$

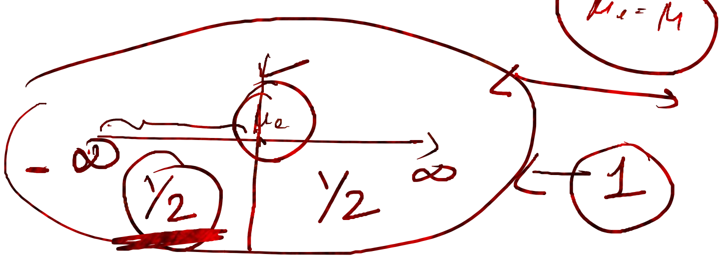
$$= \frac{\mu}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}t^2} dt$$

[Putting $t^2/2 = y$ and differentiation and using formula $\int_0^\infty e^{-t^2/2} dt = \sqrt{\pi/2}$]

$$\mu \times 1 = \mu = E(x)$$ (mean of ND is μ)

$\mu_e = \mu$

Proved.



Let us denote μ_e as median, Thus

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\mu_e} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1/2$$

$x = -\infty \Rightarrow t = -\infty$
$x = \mu_e \Rightarrow t = \frac{\mu_e - \mu}{\sigma}$

Let $t = \frac{x-\mu}{\sigma}$
$dx = \sigma dt$

or, $$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\mu_e-\mu}{\sigma}} e^{-t^2/2} dt = 1/2$$

or, $$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-t^2/2} dt = 1/2$$

$\left[\because \int_{-\infty}^{0} e^{-t^2/2} dt = \int_0^\infty e^{-t^2/2} dt = \sqrt{\pi/2}\right]$

Hence $\frac{\mu_e - \mu}{\sigma} = 0$

$\Rightarrow \sigma \neq 0 \therefore \mu_e - \mu = 0$
$\Rightarrow \mu_e = \mu = \underline{\text{mean}}$

for a normal distribution mean = median = μ (ans)

ND is symmetrical, its odd order central moment = 0.

rth order central moment, $\boxed{2r+1}$ = odd

$$\mu_r = \frac{1}{n}\Sigma(x_i-\bar{x})^r = E\left[(x-E(x))^r\right]$$

$$\mu_{2r+1} = E(x-\mu)^{2r+1}$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2r+1} f(x)\,dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2r+1} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \int_{-\infty}^{\infty} (t)^{2r+1} \frac{\sigma^{2r+1}}{\sqrt{2\pi}} e^{-t^2/2} dt \quad \left[\text{let } t = \left(\frac{x-\mu}{\sigma}\right)^2\right]$$

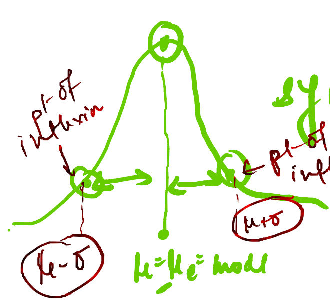
$$= \frac{\sigma^{2r+1}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{t^{2r+1} e^{-t^2/2}}_{\text{odd function}} dt$$

$$= 0$$

any odd order central moment in ND is always 0.

ND $\mu_3 = 0$ ⇒ skewness = 0 ⇒ distribution is symmetric.

(bell shaped frequency curve)



symmetric distribution. γ_1 = skewness = 0 ✓.

ND has 2 pt of inflexion.

✓ $\frac{d^2 f(x)}{dx^2} = 0$

$-\frac{1}{2}(x-\mu)^2$

$$\Rightarrow \frac{d^2 f}{dx^2} = 0$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f'(x) = ?$$

$$f''(x) = ? = 0$$

after solving

$$\Rightarrow \boxed{x = \mu \pm \sigma} \text{ (ans)}$$