07 June 2023 12:19

If $[x]$ denotes the largest integer less than or equal to x, then

gint function

$$\left[(9+\sqrt{80})^{20}\right]$$

equals

(a) $(9+\sqrt{80})^{20}-(9-\sqrt{80})^{20}$.
(b) $(9+\sqrt{80})^{20}+(9-\sqrt{80})^{20}-20$.
(c) $(9+\sqrt{80})^{20}+(9-\sqrt{80})^{20}-1$.
(d) $(9-\sqrt{80})^{20}$.

$(a+\sqrt{b})^n \rightarrow$ Integer part.

$c+\sqrt{d} \rightarrow \boxed{[\text{integer}]} + \{\text{fraction}\}$

$m < (a-\sqrt{b}) < m+1$.

$k < (a-\sqrt{b})^n < k+1$

$(a+\sqrt{b})^n + (a-\sqrt{b})^n < (a+\sqrt{b})^n + k + 1$

$\boxed{(a+\sqrt{b})^n} > \boxed{(a+\sqrt{b})^n + (a-\sqrt{b})^n - k - 1}$

integer

$\boxed{9-1 < \sqrt{80}} < 9$

$9-\sqrt{80} > 0$.

$9-\sqrt{80} < 1$.

$0 < 9-\sqrt{80} < 1$

$\boxed{0 < (9-\sqrt{80})^{20} < 1}$

$(9+\sqrt{80})^{20} + (9-\sqrt{80})^{20} < (9+\sqrt{80})^{20} + 1$

$(9+\sqrt{80})^{20} > \boxed{(9+\sqrt{80})^{20} + (9-\sqrt{80})^{20} - 1}$

INTEGER

$a \in \mathbb{Z}$. $(a+\sqrt{b})^{2n} + (a-\sqrt{b})^{2n}$

$\Downarrow$

INTEGER.

$(a+\sqrt{b})^2 + (a-\sqrt{b})^2 \rightarrow$ INTEGER.

Suppose $a, b, c \in \mathbb{R}$ and

$$f(x) = ax^2 + bx + c, x \in \mathbb{R}$$

If $\boxed{0 \le f(x) \le (x-1)^2}$ for all x, and $f(3) = 2$, then

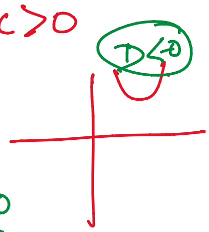
$\rightarrow$ Sandwich function

(a) $a=\frac{1}{2}, b=-1, c=\frac{1}{2}$
(b) $a=\frac{1}{3}, b=-\frac{1}{3}, c=0$.
(c) $a=\frac{2}{3}, b=-\frac{5}{3}, c=1$
(d) $a=\frac{3}{4}, b=-2, c=\frac{5}{4}$.

$0 \le f(x) \le (x-1)^2$

$ax^2+bx+c > 0$

$D < 0$

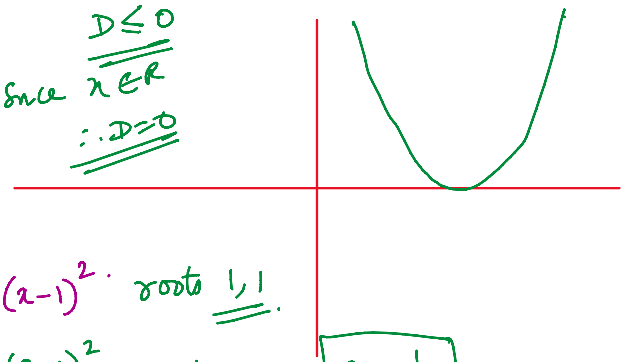


$ax^2+bx+c \ge 0$

$D \le 0$

Since $x \in \mathbb{R}$

$\therefore D = 0$



$x = 1$

$0 \le f(1) \le 0$.

$\Downarrow$

$f(1) = 0$

$f(x) = a(x-1)^2$. roots 1, 1

$2 = a(3-1)^2$ $\quad 4a = 2$ $\quad \boxed{a = \frac{1}{2}}$

$f(x) = ax^2 - 2ax + a$

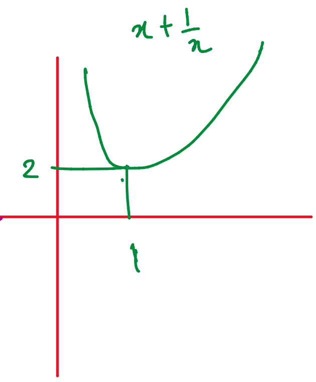
$f(x) = ax^2 + bx + c$

$\boxed{b = -2 \times \frac{1}{2} = -1}$ $\quad \boxed{c = \frac{1}{2}}$

Let $\{u_n\}_{n\geq 1}$ be a sequence of real numbers defined as $u_1 = 1$ and

$$u_{n+1} = u_n + \frac{1}{u_n} \text{ for all } n \geq 1.$$

Prove that $u_n \leq \frac{3\sqrt{n}}{2}$ for all n.



$n=1$ $\quad u_2 = u_1 + \frac{1}{u_1} = 2.$ $\quad u_1 = 1$ for $n=1$ ✓

$1 < \frac{3}{2}\sqrt{1}$ ✓

for $n=2$ ✓ $\quad u_2 = 2.$

$2 < \frac{3}{2}\sqrt{2}.$ ✓

let us assume that $u_n \leq \frac{3}{2}\sqrt{n}$ for $n>2$

$$u_{n+1} = f(u_n) \leq f\left(\frac{3}{2}\sqrt{n}\right) = \frac{3}{2}\sqrt{n} + \frac{1}{\frac{3}{2}\sqrt{n}} = \frac{3}{2}\sqrt{n} + \frac{2}{3\sqrt{n}}$$

$$u_{n+1} = \frac{9n+4}{6\sqrt{n}}.$$

$$u_{n+1} \leq \frac{3}{2}\sqrt{n+1}$$

or $\quad \frac{9n+4}{\cancel{6}_3\sqrt{n}} \leq \frac{3}{\cancel{2}}\sqrt{n+1}$

or $\quad 9n+4 \leq 9\sqrt{n(n+1)}$

or $\quad 81n^2 + 72n + 16 \leq 81n^2 + 81n.$

or $\quad 9n \geq 16$

or $\quad n \geq \frac{16}{9} \quad \Rightarrow$ the inequality is valid for $n \geq 2$

A number when divided by a divisor leaves a remainder of 27. Twice the number divided by the same divisor leaves a remainder of 3. Find the divisor?

$N = DQ + R. \quad 0 \leq R < D$

$N = DQ_1 + 27. \quad — (1)$ $\qquad 2N = \boxed{DQ_2} + 3 \; — (2)$

$N = DQ_1 + 27$ — (1)

$2N = \boxed{DQ_2} + 3$ — (2)

$DQ = D.2Q_1 + 51$

$3 \quad 17 \quad 51$

$\Downarrow$

$2N = D.(2Q_1) + 54$

$2N = \boxed{D(2Q_1) + 51} + 3.$

Divisor = 51

If a, b, c and d are four positive real numbers such that $\boxed{abcd = 1}$, what is the minimum value of (1 + a)(1 + b)(1 + c)(1 + d).

$\Downarrow$

16

$\boxed{AM \geq GM}$

$\ldots -2 \quad -1 \quad 0 \quad 1 \quad 2 \ldots$

$1+a \geq 2\sqrt{a}$

$1+b \geq 2\sqrt{b}.$

$1+c \geq 2\sqrt{c}$

$1+d \geq 2\sqrt{d}.$

$(1+a)(1+b)(1+c)(1+d) \geq 16\sqrt{abcd}.$

$\geq 16.$

Definition of a real no R.?

$R^2 \geq 0.$

a, b are 2 real no

$(a-b) \rightarrow$ real no

$(a-b)^2 \geq 0$

$a^2 - 2ab + b^2 \geq 0$

$a^2 + b^2 \geq 2ab.$

$\frac{a^2+b^2}{2} \geq ab.$

$\frac{a^2+b^2}{2} \geq \sqrt{a^2b^2}$

$AM(a^2, b^2) \geq GM(a^2, b^2)$

Let a, b, c and d be four non-negative real numbers where $a + b + c + d =$ 1. The number of different ways one can choose these numbers such that $a^2 + b^2 + c^2 + d^2 = \max\{a, b, c, d\}$ is

$\geq 0.$

(a) 1;

(b) 5;

(c) 11;

(d) 15;

$a + b + c + d = 1$

a, b, c, d.

1 | 2 max | 3 max | 4 max (equal)

1 max	2 max	3 max	4 max (equal)	
${}^4C_1 = 4$	${}^4C_2 = 6$	${}^4C_3 = 4$	${}^4C_4 = 1$	$= 15$

The polynomial $x^4 + 4x + c = 0$ has at least one real root if and only if

(a) $c < 2$;

(b) $c \leq 2$;

(c) $c < 3$;

(d) $c \leq 3$.

$x^4 = -4x - c = y$

$y = x^4$

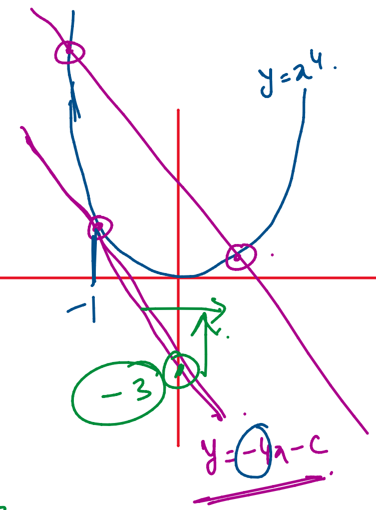
$\boxed{y = -4x - c}$

touch or intersect at least at one point

for $x = -1$

$1 - 4 + c = 0$

$c = 3$



If $c \leq 3$

$-c \geq -3$

If $y = -4x - c$ touches $y = x^4$ at one point then it will be the tangent at that point

$\therefore$ slope of the tangent $=$ slope of the line

$\frac{dy}{dx} = 4x^3$ $4x^3 = -4$

$x^3 = -1$

$\boxed{x = -1}$ → one solution

The number of all integer solutions of the equation $x^2 + y^2 + x - y = 2021$ is

(a) 5;

(b) 7;

(c) 1;

(d) 0;

$$x - y = n.$$

$$x = y + n,$$

$$(y+n)^2 + y^2 + n = 2021$$

$$y^2 + 2yn + n^2 + y^2 + n = 2021$$

$$2y^2 + 2yn + n(n+1) = 2021$$

even even even

even $\neq$ odd

$a^2 + b^2 = c^2$, $c - b = 1 \longrightarrow \boxed{b = c-1}$

a) $\boxed{a \text{ is odd}}$ b) $\boxed{b \text{ is divisible by } 4}$ c) $a^b + b^a$ is div by c.

$$a^b + b^a = a^b + (c-1)^a = a^b + c^a + a\cdot(-1)c^{a-1} + \frac{a(a-1)}{2}c^{a-2} \ldots\ldots (-1)^a.$$

$\downarrow$ $-1.$

$$= (a^b - 1) + \underbrace{\left[c^a - a c^{a-1} + \frac{a(a-1)}{2} c^{a-2} \ldots\ldots + \binom{a}{a-1} c\right]}_{\text{divisible by } c}$$

To prove $a^b - 1$ is divisible by c.

$a^2 = 2c - 1$

$b = 4m$

$$a^b - 1 = a^{4m} - 1 = (a^2)^{2m} - 1 = (2c-1)^{2m} - 1$$

$$= \left[(2c)^{2m} + (2m)(2c)^{2m-1} + \ldots\ldots + 1\right] - 1$$

$$= \underbrace{\left[(2c)^{2m} + (2m)(2c)^{2m-1} + \ldots + \binom{2m}{2m-1}(2c)\right]}_{\text{divisible by } c} + \cancel{1} - \cancel{1}$$