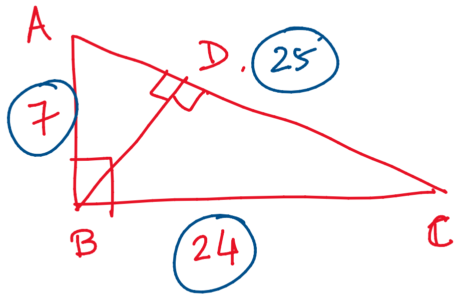
Geometry.

ABC is a right-angled triangle with the right angle at B. If $AB = 7$ and $BC = 24$, then the length of the perpendicular from B to AC is

(a) 12.2
(b) 6.72
(c) 7.2
(d) 3.36



$\frac{1}{2} AB \cdot BC = \frac{1}{2} \cdot AC \cdot BD$

$BD = \frac{AB \cdot BC}{AC}$

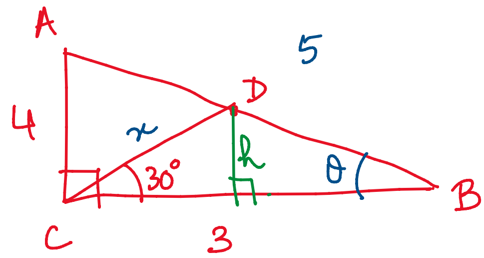
$= \frac{7 \times 24 \times 4}{25 \times 4}$

$= 6.72$

Let ABC be a right angled triangle with $BC = 3$ and $AC = 4$. Let D be a point on the hypotenuse AB such that $\angle BCD = 30°$. The length of CD is

(A) $\frac{24}{3+4\sqrt{3}}$,
(B) $\frac{3\sqrt{3}}{2}$
(C) $6\sqrt{3} - 8$
(D) $\frac{25}{12}$.

$CD = x$.



$\cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$

$h = BD \sin\theta$

$h = CD \sin 30°$

$BD \sin\theta = x \times \frac{1}{2}$

$BD \times \frac{4}{5} = \frac{x}{2}$

$\boxed{BD = \frac{5}{8}x}$

Cosine rule

$CD^2 = 3^2 + BD^2 - 2 \times 3 \times BD \cos\theta$

$x^2 = 9 + BD^2 - 6 \times BD \times \frac{3}{5}$

$x^2 = 9 + \frac{25}{64}x^2 - \frac{18}{\cancel{5}} \times \frac{\cancel{5}}{8} x$

$64x^2 = 576 + 25x^2 - 144x$.

$39x^2 + 144x - 576 = 0$.

$x = \frac{-144 + \sqrt{144^2 + 4 \times 39 \times 576}}{2 \times 39}$

$x = \frac{-144 + \sqrt{144^2 + 4 \times 4 \times 144 \times 39}}{2 \times 39}$

$= \frac{-144 + 12 \times 4\sqrt{9 + 39}}{2 \times 39} = \frac{-72 + 24\sqrt{48}}{39} = \frac{96\sqrt{3} - 72}{39}$

$= 24(4\sqrt{3} - 3) = 24(\cancel{39})$

$$\overline{}$$
$$2\times 39$$

$$= \frac{24\,(4\sqrt{3}-3)}{39}\cdot\frac{39}{39} = \frac{24\,(\cancel{39})}{\cancel{39}\,(4\sqrt{3}+3)}$$

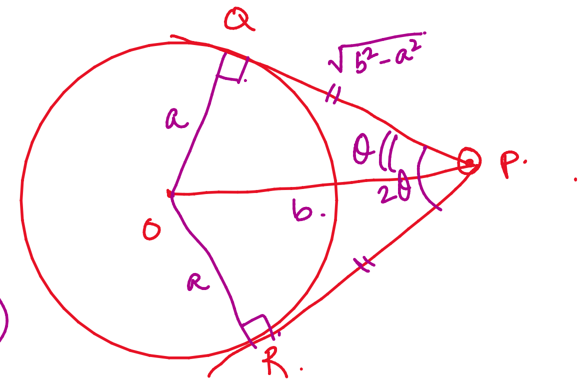
Consider a circle of radius a. Let P be a point at a distance $b(>a)$ from the center of the circle. The tangents from the point P to the circle meet the circle at Q and R. Then the area of the triangle PQR is

(A) $\frac{a\left(b^{2}-a^{2}\right)^{3/2}}{b^{2}}$;

(B) $\frac{a^{2}\sqrt{b^{2}-a^{2}}}{b}$;

(C) $\frac{b^{2}\sqrt{b^{2}-a^{2}}}{a}$;

(D) $\frac{b\left(b^{2}-a^{2}\right)^{3/2}}{a^{2}}$



area of $\Delta PQR = \frac{1}{2}\cdot PQ\cdot PR\sin(2\theta)$

$= \frac{1}{\cancel{2}}\cdot PQ^2\cdot\cancel{2}\sin\theta\cos\theta$

$= (b^2-a^2)\cdot\frac{a}{b}\cdot\frac{\sqrt{b^2-a^2}}{b} = \frac{a(b^2-a^2)^{3/2}}{b^2}$

Let a, b and c be the sides of a right-angled triangle, where a is the hypotenuse.

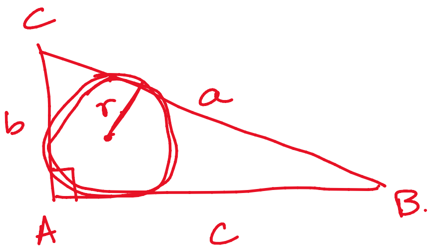
Let d be the diameter of the inscribed circle. Then

(A) $d+a=b+c$;

(B) $d+a<b+c$;

(C) $d+a>b+c$;

(D) none of the above relations need always be true.



inradius $= r = \frac{\text{area of the }\Delta}{\text{semiperimeter}}$

$s = \frac{a+b+c}{2}$.

area of $\Delta = \frac{1}{2}bc$.

$r = \frac{\frac{1}{2}bc}{\frac{a+b+c}{2}} = \frac{bc}{a+b+c}$.

$d = \frac{2bc}{a+b+c}$.

$d+a = \frac{2bc}{a+b+c} + a = \frac{2bc+a^2+ab+ac}{a+b+c} = \frac{2bc+b^2+c^2+a(b+c)}{a+b+c}$

$= \frac{(b+c)^2+a(b+c)}{(a+b+c)} = \frac{(b+c)\cancel{(a+b+c)}}{\cancel{(a+b+c)}} = b+c$

$$= \frac{(b+c)^2 + a(b+c)}{(a+b+c)} = \frac{(b+c)\cancel{(a+b+c)}}{\cancel{(a+b+c)}} = b+c$$

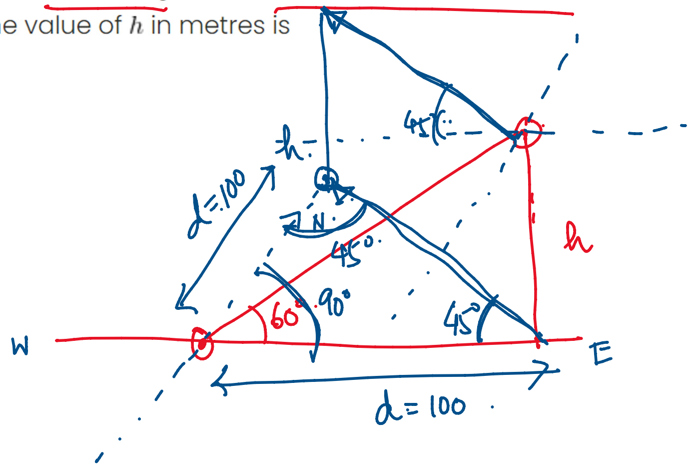
A man standing at a point O finds that a balloon at a height h metres due east of him has an angle of elevation $60°$. He walks due north while the balloon moves north-west ($45°$ west of north) remaining at the same height. After he has walked 100 metres the balloon is vertically above him. Then the value of h in metres is

(A) 50;

(B) $50\sqrt{3}$

(C) $100\sqrt{3}$; ✓

(D) $\frac{100}{\sqrt{3}}$

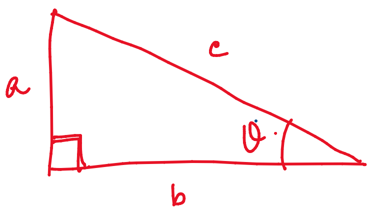


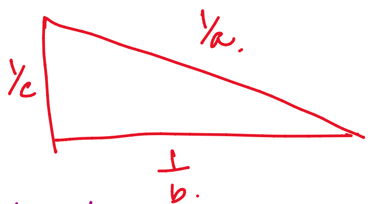
$$\frac{h}{100} = \tan 60° = \sqrt{3}.$$

$$h = 100\sqrt{3}$$

Let a, b and c be the sides of a right angled triangle. Let θ be the smallest angle of this triangle. If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are also the sides of a right angled triangle then show that $\sin\theta = \frac{\sqrt{5}-1}{2}$

$c > b > a$ $\quad$ $a^2 + b^2 = c^2$ $\quad$ $b^2 = c^2 - a^2$





$0 < \theta < 90°$

$\downarrow$

$0 < x < 1$

$x^2 < 1$

$\sin\theta = \frac{a}{c} = x$

$$\frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a^2}$$

$$\frac{b^2+c^2}{b^2c^2} = \frac{1}{a^2} \Rightarrow \frac{b^2+c^2}{b^2} = \frac{c^2}{a^2}$$

$$\frac{a^2}{\ldots} = \frac{c^2 - a^2}{\ldots} = 1 - (a^2/c^2)$$

$0 < x^2 < 1$

$$\frac{a^2}{c^2} = \frac{c^2-a^2}{2c^2-a^2} = \frac{1-(a^2/c^2)}{2-(a^2/c^2)}$$

$$\frac{b^2+c^2}{b^2c^2} = \frac{1}{a^2} \Rightarrow \frac{b^2+c^2}{b^2} = \frac{c^2}{a^2}.$$

$$\frac{a^2}{c^2} = \frac{b^2}{b^2+c^2}. = \frac{c^2-a^2}{2c^2-a^2}$$

$$x^2 = \frac{1-x^2}{2-x^2}.$$

$$2x^2 - x^4 = 1 - x^2$$

$$x^4 - 3x^2 + 1 = 0.$$

$$x^2 = \frac{3 \pm \sqrt{9-4}}{2}. = \frac{3 \pm \sqrt{5}}{2}.$$

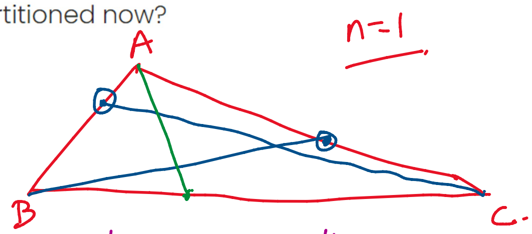
$$x^2 = \frac{3-\sqrt{5}}{2}. = \frac{6-2\sqrt{5}}{4} = \frac{(\sqrt{5}-1)^2}{2^2}$$

$$a = \frac{\sqrt{5}-1}{2}.$$

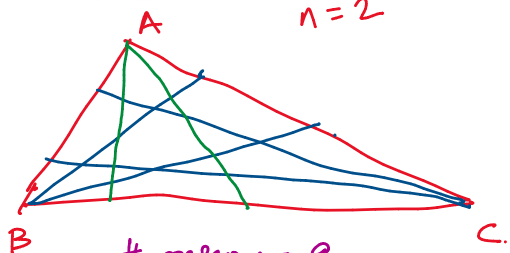
$$x = \frac{\sqrt{5}-1}{2}.$$

Let ABC be a triangle. Take n point lying on the side AB (different from A and B) and connect all of them by straight lines to the vertex C. Similarly, take n points on the side AC and connect them to B. Into how many regions is the triangle ABC partitioned by these lines? Further, take n points on the side BC also and join them with A. Assume that no three straight lines meet at a point other than A, B and C. Into how many regions is the triangle ABC partitioned now?

$n=1$



$n=2$



regions $= 4 = (1+1)^2$

regions $= 9. = (2+1)^2$

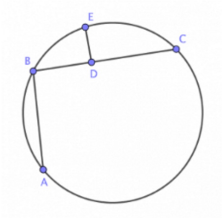
regions $= 7. \downarrow 2^3 - 1^3.$

for n points # regions $= (n+1)^2.$

for n points # regions $= (n+1)^3 - n^3.$

regions $= 19 \downarrow 3^3 - 2^3$

In the figure below, E is the midpoint of the arc $ABEC$ and the segment ED is perpendicular to the chord BC at D. If the length of the chord AB is l_1, and that of the segment BD is l_2, determine the length of DC in terms of l_1, l_2.



Let A, B and C be three points on a circle of radius 1.

(a) Show that the area of the triangle ABC equals $\frac{1}{2}(\sin(2\angle ABC) + \sin(2\angle BCA) + \sin(2\angle CAB))$

(b) Suppose that the magnitude of $\angle ABC$ is fixed. Then show that the area of the triangle ABC is maximized when $\angle BCA = \angle CAB$

(c) Hence or otherwise, show that the area of the triangle ABC is maximum when the triangle is equilateral.

In a triangle ABC, angle A is twice the angle B. Then which of the following has to be true?

(A) $a^2 = b(b+c)$

(B) $b^2 = a(a+c)$

(C) $c^2 = a(a+b)$

(D) $ab = c(a+c)$

Consider a triangle ABC with the sides a, b, c in A.P. Then the largest possible value of the angle B is

(A) 60°

(B) $67\frac{1}{2}^{\circ}$;

(C) 75°

(D) $82\frac{1}{2}^{\circ}$.

Consider a circle with centre O. Two chords AB and CD extended intersect at a point P outside the circle. If $\angle AOC = 43^{\circ}$ and $\angle BPD = 18^{\circ}$, then the value of $\angle BOD$ is

(A) 36°

(B) 29°

(C) 7°

(D) 25°,

Consider a triangle ABC. The median AD meets the side BC at the point D. A point E on AD is such that $AE : DE = 1 : 3$. The straight line BE extended meets the side AC at a point F. Then $AF : FC$ equals

(A) $1 : 6$;
(B) $1 : 7$;
(C) $1 : 4$;
(D) $1 : 3$.