

Combined Mean for two sets of data.

$$\left. \begin{array}{l} \text{set 1: } \bar{x}_1 \quad n_1 \\ \text{set 2: } \bar{x}_2 \quad n_2 \end{array} \right\} \text{Total obs } N = n_1 + n_2$$

$$\text{then Grand mean } \bar{X} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\text{or, } N \bar{X} = n_1 \bar{x}_1 + n_2 \bar{x}_2$$

General formula $N \bar{X} = \sum_{i=1}^m n_i \bar{x}_i$

$$(n_1 + n_2 + n_3 + \dots + n_m) \bar{X} = n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3 + \dots + n_m \bar{x}_m$$

Q The mean monthly salary paid to all employees in a certain company was Rs 500. \bar{X}
 The mean monthly salaries paid to male and female employees were Rs 520 and Rs 420 resp.
 Obtain the % of male to female employees in the company.

We have, salary of all employees (the grand mean) $\bar{X} = 500$.

Mean^{salary} of male employees, $\bar{x}_1 = \text{Rs } 520$.

Mean salary of female employees, $\bar{x}_2 = 420$.

Let no. of male & female employees be ' n_1 ' & ' n_2 ' resp.

$$\therefore \bar{X} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$500 = \frac{n_1 \cdot 520 + n_2 \cdot 420}{n_1 + n_2}$$

$$\text{or, } 500 = \frac{n_1 \cdot 520 + n_2 \cdot 420}{n_1 + n_2}$$

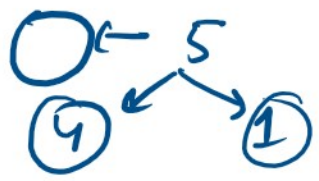
$$\text{or, } n_1 \cdot 500 + n_2 \cdot 500 = n_1 \cdot 520 + n_2 \cdot 420$$

$$\text{or, } n_2 (500 - 420) = n_1 (520 - 500)$$

$$\text{or, } n_2 \cdot 80 = n_1 \cdot 20$$

$$\text{or, } \frac{n_1}{n_2} = \frac{80}{20} = \frac{4}{1}$$

$$n_1 : n_2 = 4 : 1$$

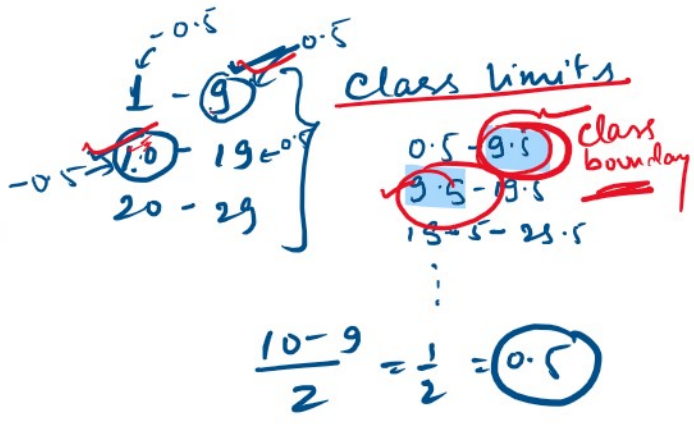


$$\% \text{ of male employees} = \frac{4}{5} \times 100 = 80\%$$

$$\% \text{ of female employees} = 20\% \left(\text{or } \frac{1}{5} \times 100 = 20\% \right)$$

Cumulative frequency (CF)

Class limits	frequency	Cumulative frequency	
		Less than	More than
15-19	37	37	200
20-24	81	37+81=118	163
25-29	43	118+43=161	82
30-34	24	161+24=185	39
35-39	9	185+9=194	15
40-44	6	194+6=200	6
$\Sigma f_i = N = 200$			



15, 16, 17, 18, 19, 20, 21, ...

Cumulative frequency

$\sum f_i = N = 200$

②

Income
220 - 249
250 - 279
280 - 309
310 - 339
340 - 369
370 - 399
400 - 419

Income	Frequency	Cummulative frequency less than	Cummulative frequency more than
220 - 249	6	6	45
250 - 279	8	6 + 8 = 14	39
280 - 309	12	26	31
310 - 339	5	31	19
340 - 369	5	36	14
370 - 399	4	40	9
400 - 419	5	45 = N	5

N = 45 (Total frequency)

Q Find the missing frequency when it is known that AM = 11.09.

Class limits	9.3 - 9.7	9.8 - 10.2	10.3 - 10.7	10.8 - 11.2	Total
Frequency	2	5	f_3	f_4	60
	11.3 - 11.7	11.8 - 12.2	12.8 - 13.2		
	6	3	1		

Solution

Class limits	Frequency (f)	Mid value x	fx
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9.3 - 9.7	2	9.5	19.0
9.8 - 10.2	5	10.0	50
10.3 - 10.7	f_3	10.5	$10.5f_3$
10.8 - 11.2	f_4	11.0	$11f_4$
11.3 - 11.7	14	11.5	161
11.8 - 12.2	6	12.0	72
12.3 - 12.7	3	12.5	37.5
12.8 - 13.2	1	13.0	13
	$\Sigma f = 60$		$\Sigma xi \cdot fi = 352.5$ $+ 10.5f_3 + 11f_4$

Total frequency, $\Sigma f = 60$

or, $f_3 + f_4 + 31 = 60$

or, $f_3 + f_4 = 29$ — (1)

A.M, $\bar{x} = 11.09$

$\frac{1}{N} \Sigma xi \cdot fi = 11.09$

$\frac{352.5 + 10.5f_3 + 11f_4}{60} = 11.09$

or, $10.5f_3 + 11f_4 = 665.4 - 352.5$
 $10.5f_3 + 11f_4 = 312.9$ — (2)

$(f_3 + f_4) \times 11 = 29 \times 11$

$10.5f_3 + 11f_4 = 312.9$

$11f_3 + 11f_4 = 319$

$-0.5f_3 = -6.1$

$\frac{319.0}{312.9} = 6.1$

$f_3 = \frac{6.1}{0.5} = 12.2$

$f_3 = 12.2$

$$f_3 = 12.2$$

$$\text{then } f_4 = 29 - f_3 = 29 - 12.2$$

$$f_4 = 16.8$$

Geometric Mean (G.M)

(i) without frequency let the set of observations be: $x_1, x_2, x_3, \dots, x_n$
no. of 'n'

$$\text{then } G.M = \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$

$$G.M = (x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n)^{1/n}$$

(ii) with frequency x_1, x_2, \dots, x_n with frequencies
 f_1, f_2, \dots, f_n

$$\text{then, } G.M = \left(x_1^{f_1} \cdot x_2^{f_2} \cdot x_3^{f_3} \cdot \dots \cdot x_n^{f_n} \right)^{1/N}$$

$$\text{where } N = \sum f_i = f_1 + f_2 + \dots + f_n$$

Properties: ① if all observations are same, then GM of all obs is same value.

that is let $x_i = c \quad \forall i = 1(1)n$

$[c, c, c, \dots, c]$
(n times)

$$\text{then } G.M = [c \cdot c \cdot c \cdot \dots \cdot c]^{1/n}$$
$$= (c^n)^{1/n} = c$$

∴ 1. algorithm of GM of a set of observations is 1.7

② The logarithm of GM of a set of observations is equal to A.M of the log of all observations (ie $\log(GM) = \frac{1}{n} \sum \log(x_i)$)

$$GM = (x_1 \cdot x_2 \cdots x_n)^{1/n}$$

$$\log(GM) = \frac{1}{n} \log(x_1 \cdot x_2 \cdots x_n)$$

$$= \frac{1}{n} [\log x_1 + \log x_2 + \log x_3 + \cdots + \log x_n]$$

$$\log GM = \frac{1}{n} \sum_{i=1}^n \log(x_i)$$

(Proved)

③ Harmonic Mean

① without f let the 'n' observations be x_1, x_2, \dots, x_n

then H.M = $\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$

② with f

let the 'n' observations be x_1, x_2, \dots, x_n with corresponding frequencies f_1, f_2, \dots, f_n

then, H.M = $\frac{\sum f_i}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_n}{x_n}} = \frac{N}{\sum_{i=1}^n \frac{f_i}{x_i}}$

Relation between A.M, G.M and H.M.

$A.M \geq G.M \geq H.M.$ } imp.

① $A \cdot M \geq G \cdot M \geq H \cdot M.$

② $A \cdot M \times G \cdot M = (HM)^2$

Imp.