Combined Mean for two sets of data.

set 1 : $\bar{x}_1$ n_1
set 2 : $\bar{x}_2$ n_2

Total obs $N = n_1 + n_2$

then Grand mean $\bar{X} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$

or, $N\bar{X} = n_1\bar{x}_1 + n_2\bar{x}_2$

General formula $N\bar{X} = \sum_{i=1}^{m} n_i\bar{x}_i$

$(n_1+n_2+n_3+\cdots+n_m)\bar{X} = n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3 + \cdots + n_m\bar{x}_m$

Q The mean monthly salary paid to all employees in a certain company was Rs 500. $\bar{X}$
The mean monthly salaries paid to male and female employees were $\bar{x}_1$ Rs 520 and Rs 420 $\bar{x}_2$ resp.
Obtain the % of male to female employees in the company.
n_1 n_2

We have, salary of all employees (the grand mean) $\bar{X} = 500$.

Mean salary of male employees, $\bar{x}_1 = $ Rs 520.
Mean salary of female employees, $\bar{x}_2 = 420$.
Let no. of male & female employees be 'n_1' & 'n_2' resp.

$$\therefore \bar{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$$

$500 = n_1 \cdot 520 + n_2 \cdot 420$

$$n_1+n_2$$

or, $$500 = \frac{n_1\,520 + n_2\,420}{n_1+n_2}$$

or, $$n_1 500 + n_2 500 = n_1 520 + n_2 420$$

or, $$n_2(500-420) = n_1(520-500)$$

or, $$n_2\,80 = n_1\,20$$

or, $$\frac{n_1}{n_2} = \frac{80}{20} = \frac{4}{1}$$

$$n_1 : n_2 = 4 : 1$$

5

4 1

% of male employees $= \frac{4}{\cancel{5}} \times \cancel{100}^{20}$ $= 80\%$

% of female employees $= 20\%$ $\left(\text{or } \frac{1}{5} \times 100 = 20\%\right)$.

Cummulative frequency (CF)

Class limits	frequency	Cummulative frequency: Less than	Cummulative frequency: More than
15-19	37	37	200
20-24	81	37+81 = 118	163
25-29	43	118+43 = 161	82
30-34	24	161+24 = 185	39
35-44	9	185+9 = 194	15
45-59	6	194+6 = 200	6
	$\Sigma f_i = N = 200$		

-0.5   +0.5

1 - 9

-0.5 10 - 19 +0.5

20 - 29

Class limits

0.5 - 9.5

9.5 - 19.5

19.5 - 29.5

⋮

Class boundary

$$\frac{10-9}{2} = \frac{1}{2} = 0.5$$

15, 16, 17, 18, 19, 20, 21 . . . .

Cummulative frequen

$\Sigma f_i = N = 200$

② **Income** | **Frequency** | Cummulative frequency: Less than | More than

Income	Frequency	Less than	More than
220-249	6	6	45
250-279	8	6+8=14	39
280-309	12	26	31
310-339	5	31	19
340-369	5	36	14
370-399	4	40	9
400-419	5	45 =N	5

N = 45 (Total frequency)

Q Find the missing frequency when it is known that AM = 11.09.

Class limits	9.3-9.7	9.8-10.2	10.3-10.7	10.8-11.2	11.3-11.7	11.8-12.2	12.8-13.2	Total
Frequency	2	5	f_3	f_4	6	3	1	60 N

Solution

Class limits	Frequency (f)	Mid value x	fx

9.3 - 9.7	2	9.5	19.0
9.8 - 10.2	5	10.0	50
10.3 - 10.7	f_3	10.5	$10.5f_3$
10.8 - 11.2	f_4	11.0	$11f_4$
11.3 - 11.7	14	11.5	161
11.8 - 12.2	6	12.0	72
12.3 - 12.7	3	12.5	37.5
12.8 - 13.2	1	13.0	13
	$\Sigma f = 60$		$\Sigma x_i f_i = 352.5 + 10.5f_3 + 11f_4$

Total frequency, $\Sigma f = 60$

or, $f_3 + f_4 + 31 = 60$

or, $\boxed{f_3 + f_4 = 29}$ — (1)

A.M, $\bar{x} = 11.09$

$\frac{1}{N}\Sigma x_i f_i = 11.09$

$$\frac{352.5 + 10.5f_3 + 11f_4}{60} = 11.09$$

or, $10.5f_3 + 11f_4 = 665.4 - 352.5$

$\boxed{10.5f_3 + 11f_4 = 312.9}$ — (2)

$(f_3 + f_4) \times 11 = 29 \times 11$

$$
\begin{aligned}
10.5f_3 + 11f_4 &= 312.9 \\
11f_3 + 11f_4 &= 319 \\
\hline
-0.5f_3 &= -6.1
\end{aligned}
$$

$$
\begin{array}{r}
319.0 \\
312.9 \\
\hline
6.1
\end{array}
$$

$$f_3 = \frac{6.1}{0.5} = \frac{61}{5} = 12.2$$

$\boxed{f_3 = 12.2}$

$$\boxed{f_3 = 12.2}$$

then $f_4 = 29 - f_3 = 29 - 12.2$

$$\boxed{f_4 = 16.8}$$

Geometric Mean (G.M)

(i) [without frequency] Let the set of observations be : $x_1\ x_2\ x_3 \cdots x_n$

no. of 'n'

then $G.M = \sqrt[n]{x_1 . x_2 \cdots x_n}$

or, $$\boxed{GM = (x_1 . x_2 . x_3 \cdots x_n)^{1/n}}$$

(ii) [with frequency] $x_1\ .x_2 \cdots x_n$ with frequencies

$f_1\ f_2 \cdots f_n$

then, $$\boxed{GM = \left(x_1^{f_1} . x_2^{f_2} . x_3^{f_3} \cdots x_n^{f_n}\right)^{1/N}}$$

where $N = \sum f_i = f_1 + f_2 + \cdots + f_n$

Properties: ① if all observations are same, then GM of all obs is same value.

that is let $x_i = c$ $\forall\ i = 1(1)n$ $\left[c, c, c \cdots c \text{ (n times)}\right]$

then $GM = [c.c.c \cdots c]^{1/n}$

$= (c^n)^{1/n} = c$

② The $\boxed{\text{Logarithm of GM}}$ of a set of observations is

(2) The logarithm of GM of a set of observations is equal to A.M of the log of all observation (ie $\log(GM) = \frac{1}{n}\sum \log(x_i)$)

$$GM = (x_1 \cdot x_2 \cdots x_n)^{1/n}$$

$$\log(GM) = \frac{1}{n}\log(x_1 \cdot x_2 \cdots x_n)$$

$$= \frac{1}{n}\left[\log x_1 + \log x_2 + \log x_3 + \cdots + \log x_n\right]$$

$$\log GM = \frac{1}{n}\sum_{i=1}^{n}\log(x_i)$$

(Proved)

(3) Harmonic Mean

(1) Without f

Let the 'n' observations be $x_1, x_2, \cdots, x_n$

then
$$H.M = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} 1/x_i}$$

(2) with f

Let the 'n' observations be $x_1, x_2, \cdots, x_n$ with corresponding frequencies $f_1\, f_2 \cdots f_n$

then,
$$HM = \frac{\sum f_i}{\frac{f_1}{x_1}+\frac{f_2}{x_2}+\cdots+\frac{f_n}{x_n}} = \frac{N}{\sum_{i=1}^{n}\frac{f_i}{x_i}}$$

Relation between A.M, G.M and H.M.

(1) A.M $\geq$ G.M $\geq$ H.M. } imp.

1. $A.M \geq G.M \geq H.M.$
2. $A.M \times G.M = (HM)^2$

imp.