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Linear Algebra

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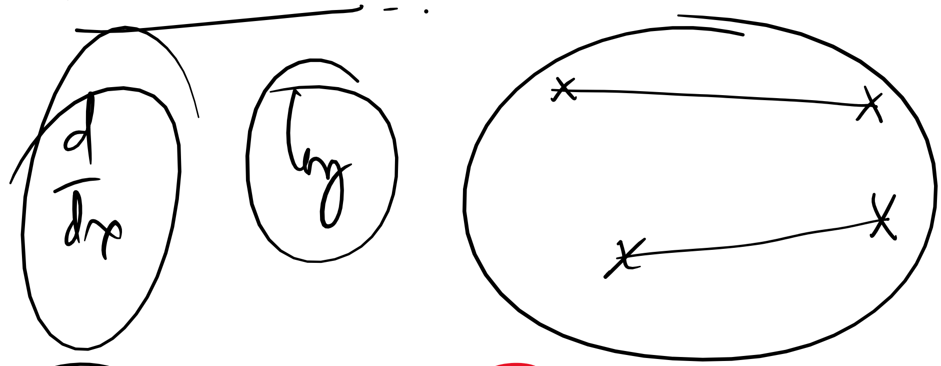
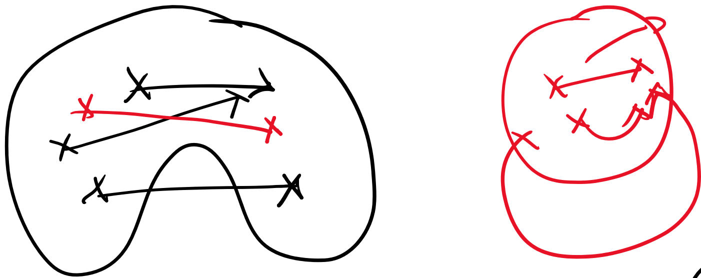


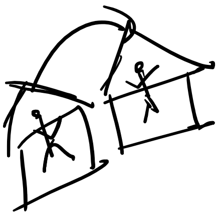


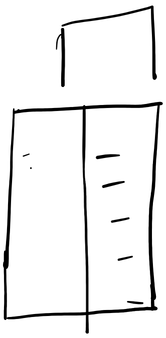
A linear operator T If $T^K = 0$

then $T^m = 0$ & $T^{m-1} \neq 0$

Nilpotent operator

Nilpotency Index





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Range of operator

Linear dependency

Linear Ind

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13

ISI M-Tech CS+CRS+QROR
ISI M-Math
M-Stat (Math)
ISI MS QE

TIFR GS maths

DS

~~UGC Net~~
CSIR

M-Stat (Maths)
ISI MS QE
ISI MSQMS
IIT
Core Maths
JAM maths + Math Stats
CMI ent — DS, Masters
~~UGC Net~~
CSIR
Math Stat
25 + Set (B)
25-34

null space

Range
Kernel
Rank
nullity

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix} \xrightarrow{C_3 - C_1} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix} C_3 - C_2$$

$(0, 1, -1)$ $(0, 2, 1)$ L.I. C.V.

Basis for Range Space $\rightarrow$ $R \Rightarrow 2$

Null Space (Kernel)

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 + v_3 = 0$

$v_1 + 2v_2 + 3v_3 = 0$

$v_1 = v_2$
$v_1 = -v_3$

$-v_1 + v_2 = 0$

$v_3 = 1$

$v_1 + v_2 \ldots$

$v_1 + 2v_2 + 3v_3 = 0$

$-v_1 + v_2 = 0$

$v_1 = 1, \quad v_2 = 1$

$v_3 = -1$

So, basis of the null space of $\{1, 1, -1\}$

nullity $A = 1$

$R + N = \cancel{\text{Columns}}$

$$\begin{pmatrix} 3 & 1 \\ -6 & -2 \end{pmatrix} \quad \det = 0$$

$R = 1$

Basis $\begin{pmatrix} 3 \\ -6 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Kernel of A is vector $\cancel{Av = 0}$

Space $\longleftrightarrow$ spanned by $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$$\boxed{R + N = \text{Dimension}}$$

Sylvester's Law of nullity

$\max \{n(A), n(B)\} \leq n(AB) \leq n(A) + n(B)$

$$\max\{n(A), n(B)\} \le n(AB) \le n(A)+n(B)$$

$n =$ nullity

for Rank

$$R(A) + R(B) - n \le R(AB) \le \min\left\{\begin{matrix} R(A), \\ R(B) \end{matrix}\right\}$$

$\rightarrow$ finite dimensional vectors

$\circledast$ $\boxed{\text{TRANSFORMATION}}$

non-singular

$T \rightarrow V(F)\ V(F)$

$T(\ker T)$ $\alpha \in U$

$T(\alpha) = 0$

Singular

$0 \neq \alpha \in U$

$T(\alpha) = 0$

$T(x, y, z) = (x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta, z)$

$T(x, y, z) = (0, 0, 0)$

$x\cos\theta - y\sin\theta = 0 = x\sin\theta + y\cos\theta = 0$

$\boxed{x=0=y}$

$(x,y,z)=(0,6,0)$ $\quad \mathbb{R}^3$

$\left(x\right.$ T is non-singular

ISI M.Math 2006

$T(x_1,x_2,x_3) = (x_1-x_2,\ x_1-x_3,\ 0)$ $\to 0 \quad \to 0 \quad \to 0$

$N(T)$ $\quad R(T)$

null space Range space

~~a)~~ $\dim N(T)=2$ $\quad$ ~~b) $\dim R(T)=2$~~

c) $R(T)=N(T)$, $\quad$ d) $N(T)\subset R(T)$

Ans
Lem

$(x_1,x_2,x_3)\in N(T)$

Putting 0 to each specific space

$x_1=x_2,\ x_1=x_3$

$\boxed{\dim N(T)=1}$

$\dim R(T)=3-1=2$

$\times$ $(1,1,1)\in N(T)$ but $(1,1,1)\notin R(T)$

(c) $\times$ $\quad$ (d) $\times$

(c) ✗ (d) ✗

(f)

$T(1,2) = (2,3)$

$T(0,1) = (1,4)$

$T(5,6) = ?$

~~Sort~~

$(5,6) = \alpha(1,2) + \beta(0,1)$

$5 = \alpha, \quad \beta = -4$

$(5,6) = 5(1,2) - 4(0,1)$

$T(5,6) = 5T(1,2) - 4T(0,1)$

$5(2,3) - 4(1,4) = (6,-1)$

New way out

Standard Basis $\{e_1, e_2, e_3\}$

of R^3 is

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

the $T = ?$

As matrix T corr to basis $\{e_1, e_2, e_3\}$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\therefore T(e_1) = e_3$, $T(e_3) = e_1$

$\boxed{T(e_2) = e_2}$,

So, maps on the subspaces ~~e_1~~ on itself

$\Rightarrow e_2$

∴ Solve CE we get E.v.

$1, 1, -1$

Span

E.vector corresponding to $\lambda = 1$

$$\begin{bmatrix} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & 2 \\ 3 & -1 & -4 \end{bmatrix}$$

CMI 2013

$\ldots 2b - 3c = 0 \quad \ldots 3$

CMI 20...

$$U = \left\{ (a,b,c) \;\middle|\; \begin{array}{l} a+2b-3c=0 \\ 2a+5b+2c=0 \\ 3a-b-4c=0 \end{array} \right\} \subset \mathbb{R}^3$$

dim Ran ?

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & 2 \\ 3 & -1 & -4 \end{bmatrix}$$

$$|A| = 1(-20-6) - 2(-8-6) - 3(-2-15)$$

$$= +(6) \neq 0$$

$R(A) = 3$

$$N = n - R(A) = 3-3 = 0$$

$$\underline{\underline{\dim(U) = 0}}$$