RISK AND UNCERTAINTY 4.4.2024

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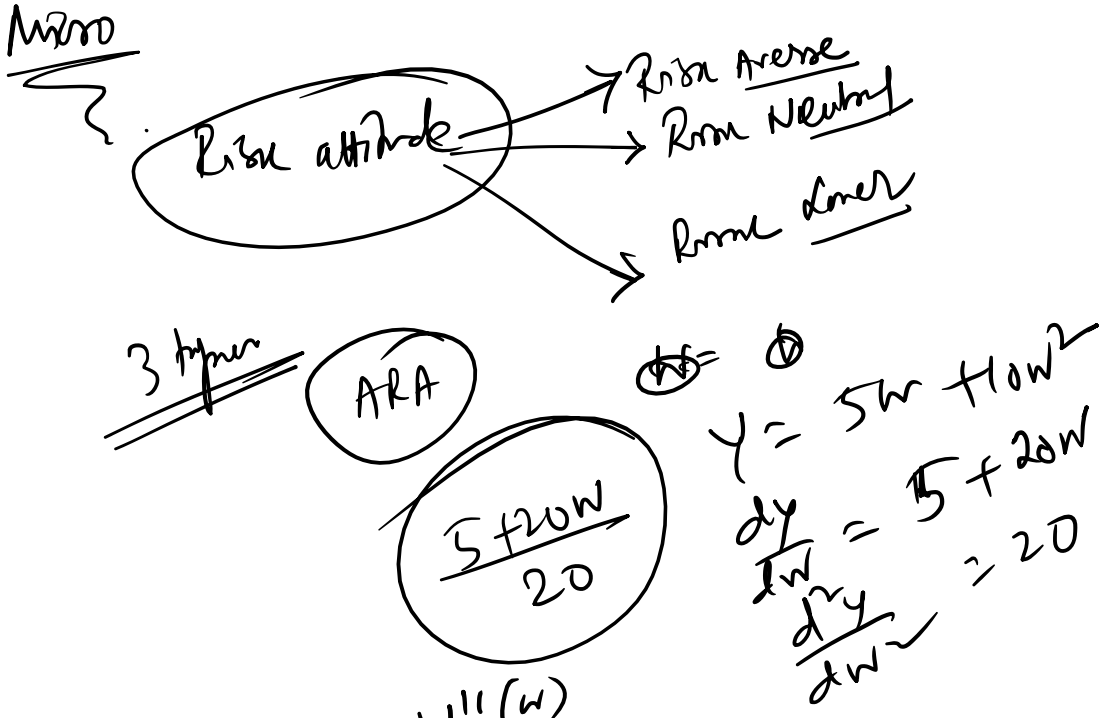

Formula $ARA = -\frac{U''(W)}{U'(W)}$

① $U = a + bW - cW^2$ $b>0, c>0$

② $U = 10\log W$

Ans ① $U = a + bW - cW^2$

$U'(W) = b - 2cW = 0$

$U''(W) = -2c < 0$ Utility is max..

$ARA = -\frac{U''(W)}{U'(W)} = -\frac{(-2c)}{(b-2cW)}$

$= \frac{2c}{b-2cW}$

$$= \frac{-}{b-2cw}$$

$$\frac{d(ARA)}{dw} = \frac{(b-2cw)0 - 2c(-2c)}{(b-2cw)^2}$$

$$= \frac{4c^2}{(b-2cw)^2} > 0$$

(ii) $U = \log W$

$U'(w) = \frac{10}{w} > 0$

$U''(w) = \frac{-10}{w^2} < 0$

$ARA = -\frac{(-10/w^2)}{10/w} = \frac{1}{w} > 0$

$\frac{d(ARA)}{dw} = -\frac{1}{w^2} < 0$

Q5) Mr X invests Rs 2500 in a risky project that yield 10000 if successful & nothing otherwise.. $U = \sqrt{W}$ find the probability that the project fails?

$p\sqrt{W_1} + (1-p)\sqrt{W_2} = U = \sqrt{W}$

or, $p\sqrt{10000} + (1-p)\sqrt{0} = \sqrt{2500}$

success

failure

$(1-p)\sqrt{0} = \sqrt{2500}$

Given

$p\sqrt{10000} + (1-p)\sqrt{0} = \sqrt{2500}$

$100p + 0 = 50$

$p = \frac{1}{2}$

Insurance Problems

Q. Consider has a initial wealth of 1,60,000 & there is a Risk of fire. There is a 5% prob of a major fire with loss of 70000, 5% bad fire loss 1,20,000. Insurance → he learns from 7620 of loss. $V = \sqrt{W}$ more premium.

So, $0.10\left(160000 - 7620 - P\right)^{\frac{1}{2}} + 0.90\left(160000 - P\right)^{\frac{1}{2}}$

$= 0.05\left(\sqrt{160000 - 70000}\right) + 0.05\left(\sqrt{160000 - 120000}\right)$

$+ 0.90\left(\sqrt{160000}\right) = 385$

$P = 11004$ more premium he is willing to pay.

Gamble Question.

$U = \sqrt{W}$

initial wealth 36 ✓

Gamble is win 13 ✓ $p = 2/3$

lose 11 $p = 1/3$

How you accept it?

$U = \sqrt{36} = 6$

$\sqrt{36 - 11}$

$U = \sqrt{36}$

Expected Utility $= \frac{2}{3}\sqrt{36+13} + \frac{1}{3}\sqrt{36-11}$

$\Rightarrow \frac{2}{3}\sqrt{49} + \frac{1}{3}\sqrt{25}$

$\Rightarrow \frac{2}{3} \times 7 + \frac{1}{3} \times 5$

$\Rightarrow \frac{19}{3} \rightarrow 6.33$

Couple — Initial wealth 100

win 20 $p = 1/2$

lose 20 $p = 1/2$

$U = W^2$

EW? on the play?

$EW = \frac{1}{2}(100+20)^2 + \frac{1}{2}(100-20)^2$

$= \frac{120^2}{2} + \frac{80^2}{2}$

$= 10400$

$U = W^2 = 10000$

Talent show problem

Sam

$U = 1 - \frac{1}{W}$

If teacher $W = 5$ with prob $= 1$

If others $W = 400$, $p = 0.01$

prob to choose $W = 2$

How much she will be willing to pay to

How much she will be willing to pay to the buyer that owners of future tender?

Ans: $U_T = 1 - \frac{1}{5} = 0.8$

dung → C

$EU_A = 0.01\left(1 - \frac{1}{400}\right) + 0.99\left(1 - \frac{1}{2}\right)$

$= 0.505$

$EU_{\text{Buyer}} = 0.01\left[1 - \frac{1}{(400 - Q)}\right] + 0.99\left[1 - \frac{1}{(5 - Q)}\right]$

Buyer Tender

$\Rightarrow 0.8 \Rightarrow EU$

Solving we get $\boxed{Q = 0.0494}$