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Equation of a plane

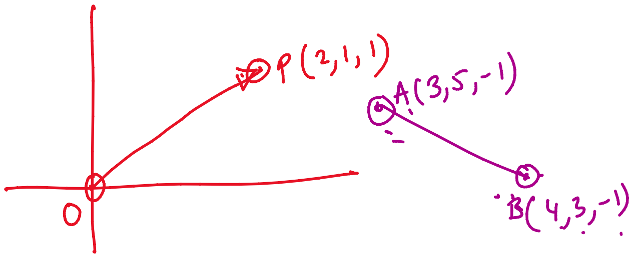
$ax + by + cz = d$.

normal vector $\vec{n} = (a, b, c)$

$d = \perp$ distance of the plane from the origin

Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, – 1), (4, 3, – 1).

line $\vec{OP} = \underbrace{(0,0,0)}_{\text{st pt}} + \lambda \underbrace{(2,1,1)}_{\text{dir. vect } (\vec{b}_1)}$



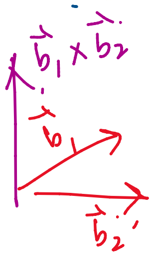
line $\vec{AB} = (3,5,-1) + k(4-3, 3-5, -1+1)$

$= \underbrace{(3,5,-1)}_{\text{st pt}} + k \underbrace{(1,-2,0)}_{\text{dir vect } (\vec{b}_2)}$

$|\vec{b}_1||\vec{b}_2| \cos\theta = \vec{b}_1 \cdot \vec{b}_2 = 0$

$(2,1,1) \cdot (1,-2,0) = 2 + (-2) + 0 = 0.$

2. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1 n_2 - m_2 n_1$, $n_1 l_2 - n_2 l_1$, $l_1 m_2 - l_2 m_1$



$L_1 : (a_1, b_1, c_1) + \lambda(l_1, m_1, n_1)$ $\vec{b}_1$

$L_2 : (a_2, b_2, c_2) + k(l_2, m_2, n_2)$ $\vec{b}_2$

$(l_1, m_1, n_1) \cdot (l_2, m_2, n_2) = 0 \quad \text{—} (1)$

$\boxed{l_1 l_2 + m_1 m_2 + n_1 n_2 = 0}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = (m_1 n_2 - m_2 n_1)\hat{i} - (l_1 n_2 - l_2 n_1)\hat{j} + (l_1 m_2 - l_2 m_1)\hat{k}$

Find the angle between the lines whose direction ratios are a, b, c and $b - c, c - a, a - b$.

$\vec{p}$ $\vec{q}$

direction ratios $= (a, b, c)$

direction cosines

$b-c,\ c-a,\ a-b$.)

$\vec{q}$ $\qquad \vec{p}$

direction ratios ...

direction cosines

$$r = \hat{a} + \lambda(\vec{b})$$

$$= \frac{a}{\sqrt{a^2+b^2+c^2}},\ \frac{b}{\sqrt{a^2+b^2+c^2}},\ \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

$|\vec{b}| = \sqrt{x^2+y^2+z^2}$. $\quad \vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$,

direction ratios

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|}$$

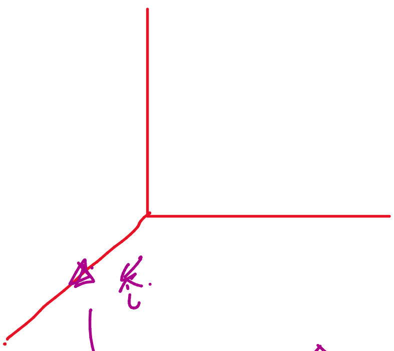
$$\hat{b} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$

$0 = \vec{p}\cdot\vec{q} = |\vec{p}||\vec{q}|\cos\theta$.

$\cos\theta = 0$.

direction cosines.

4. Find the equation of a line parallel to x-axis and passing through the origin.



$\vec{r} = \lambda a\hat{i} = x\hat{i}$

$$\frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0}.$$

$\vec{r} = (0,0,0) + \lambda(a,0,0)$

$x = 0 + \lambda a$. $\quad y = 0 \quad z = 0$

$$\frac{x-0}{a} = \lambda \qquad \frac{y-0}{0} = \lambda \qquad \frac{z-0}{0} = \lambda$$

6. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

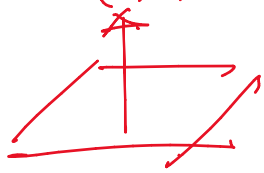
$-9k + 2k - 10 = 0$.

$k = -\frac{10}{7}$.

7. Find the vector equation of the line passing through $(1, 2, 3)$ and perpendicular to the plane $\vec{r}\cdot(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

direction vector of the normal to the plane.

$(1, 2, -5)$



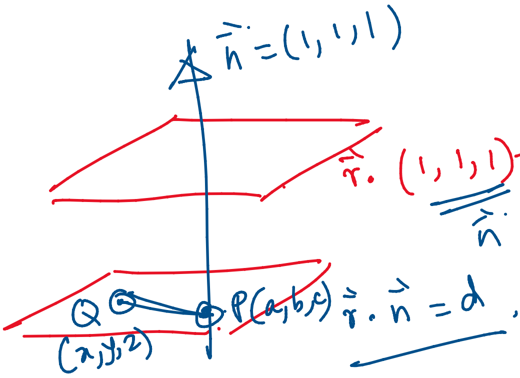
$\vec{r}\cdot\vec{n} = d$. $\rightarrow$ equ of a plane.

$\vec{r} \cdot n = d$ → equ of a plane

$(xi + yj + zk) \cdot (ai + bj + ck) = d$

$ax + by + cz = d$

8. Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$.



direction of PQ = $(x-a, y-b, z-c)$

$(x-a, y-b, z-c) \cdot (1,1,1) = 0$

concurrent forces in a plane, theory of couples, system of parallel forces.