CAT LEVEL 2 QUESTIONS (1)

Thursday, August 24, 2023 5:39 PM

P/C Problems & Solutions

CAT LEVEL 2 QUESTI...

10 In how many ways can 11 identical books on English and 9 identical books on Maths be placed in a row on a shelf so that two books on Maths may not be together?

(a) 110 (b) 220 (c) 330 (d) 440

arrange 11 Identical books → 1 way

11 Books → 12 spaces

12 spaces → 9 Books $^{12}C_9$ ways

All maths → 1 ways on arrangement

$1 \times {}^{12}C_9 \times 1 \Rightarrow {}^{12}C_9 \rightarrow \frac{12!}{9! \times 3!}$

$\Rightarrow 220$

$(11! \times 9! \times {}^{12}C_9)$

$3! \rightarrow 6$

$1 \times {}^{12}C_9 \times 1 \Rightarrow {}^{12}C_9 \rightarrow \frac{12!}{9! \times 3!}$

$(11! \times 9! \times {}^{12}C_9)$

$\Rightarrow 220$

111

I II III
II I IV
$> 3! \rightarrow 6$

Step prime factors

**11** Find the total number of factors of 1680.
(a) 40 (b) 50 (c) 60 (d) 30

$1680 = 2^4 \cdot 3^1 \cdot 5^1 \cdot 7^1$

formula

$N = a^p \, b^q \cdot c^r \ldots$

No of factors $= (p+1)(q+1)(r+1) \ldots$

$= (4+1)(1+1)(1+1)(1+1)$

$= 5 \cdot 2 \cdot 2 \cdot 2 = 40$

24

$1 \cdot 24$
$2 \cdot 12$
$3 \cdot 8$
$4 \cdot 6$

168

| | |
|---|---|
| 2 | 1680 |
| 2 | 840 |
| 2 | 420 |
| 3 | 210 |
| 2 | 70 |
| 5 | 35 |
| | 7 |

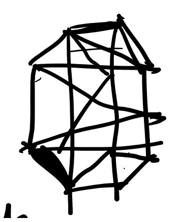
$24 = 2^3 \cdot 3$

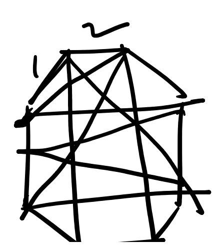
$(3+1)(2+1)$

$\Rightarrow 12$

| | |
|---|---|
| 2 | 24 |
| 2 | 12 |
| 2 | 6 |
| | 3 |

**14** The number of triangles whose vertices are at the vertices of an octagon but none of the sides of such triangles are taken from the sides of the octagon.
(a) 8 (b) 15
(c) 16 (d) none of these





Required Triangle = Total Triangle ...

(d) none of these

Required Triangle = Total Triangle − One common side − 2 common sides

origin

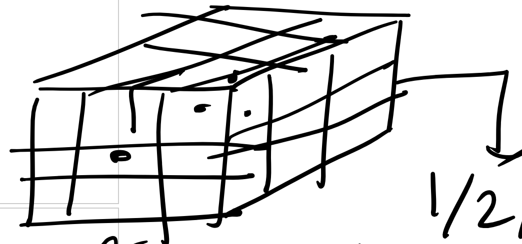
octagon → All corners

$$^8C_3 - (^8C_1 \times {^4C_1}) - 8$$

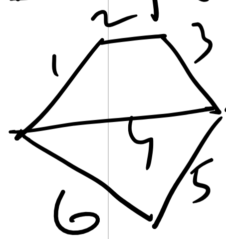
$$\Rightarrow \frac{8!}{5!3!} - (8 \times 4) - 8$$

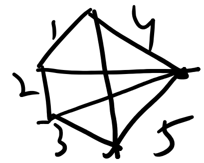
$$\Rightarrow 16$$

~~$^7C_3 - {^7C_1} - 7$~~



1/2/3 XX 4/5/6





1, 2, 3, 5

1, 2, 6, 25

The number of ways in which we can select 5 numbers from the set of numbers {1, 2, 3, …, 25} such that none of the selections includes four consecutive numbers is :

(a) 53109 (b) 13350

(c) 10035 (d) none of these

Total $^{25}C_5$

$(^{25}C_5 - 21)$

1, 2, 3, 4, 5

2, 3, 4, 5, 6

21, 22, 23, 24, 25

21

$a \geq -1$ $\quad a-1=u$ $\quad u \geq 0$ $\quad w \geq 0$

$b \geq -1$ $\quad b-1=v$ $\quad v \geq 0$ $\quad x \geq 0$

$c \geq -1$

$d \geq -1$ $\quad 1, 2, 3$

$u-1+v-1+w-1+x-1=12$

$u+v+w+x=16$

**16** The number of integral solutions for the equation $a+b+c+d=12$, where $(a, b, c, d) \geq -1$ is :

(a) ${}^{19}C_3$ (b) ${}^{18}C_4$

(c) ${}^{20}C_4$ (d) none of these

$a+b+c=3$

$3+3-1C_{3-1}$

$\Rightarrow 5C_2 = 10$

$a+b+c=5$

$5+3-1C_{3-1}$

$\frac{5!}{2!3!} = \frac{120}{12} = 10$

1,0,0 / 0,1,0 / 0,0,1

3,0,0 / 0,3,0 / 0,0,3

1,2,0 / 2,1,0

$a+b=10$

$10+2-1C_{2-1}$

$\Rightarrow 11C_1$

$\Rightarrow 11$

(r, b, g)

0, 10, 0

2, 7, 1

4, 4, 2

6, 1, 3

~~8, 1, 4~~

**19** There are three piles of identical red, green and blue balls and each pile contains atleast 10 balls. The number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected is :

(a) 1 (b) 2 (c) 4 (d) 6

Logical Cases

4 ways

$x \neq 0$

10 4

$2+2+4+2$

$2 \cdot 5 \times 4$

**21** The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question is :

(a) 11 (b) ${}^{21}C_7$ (c) 18 (d) 235

$u_i = x_i - 2$ $x_1 + x_2 + \cdots + x_8 = 30$

$x_i \geq 2$

$x_1 \geq 2$

$x_1 - 2 \geq 0$

$u_1 \geq 0$

$u_1 + u_2 + \cdots + u_8 + 16 = 30$

$\sum u_i = 46$

$u_1 + u_2 + \cdots + u_8 = 46$

4

Here, $x_i \geq 2$

$u_1 + u_2 + \cdots + u_8 = 14$

${}^{14+8-1}C_{8-1} = {}^{21}C_7$

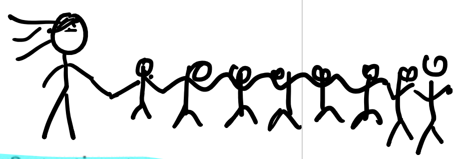
$x + y + z = 6$

${}^{6+3-1}C_{3-1} \Rightarrow {}^{8}C_2 = 28$

**26** The total number of ways of selecting 6 coins out of 10 one rupee coins, 6 fifty paise coins and 8 twenty paise coins is :

(a) 28 (b) 14 (c) 13 (d) 19

Poor Nargis



**27** Nargis has 8 children and she takes 3 at a time to children's park as often as she can without taking the same 3 children together more than once. The number of times she will go to the park is :

(a) 56 (b) 14 (c) 28 (d) 76

$^{8}C_{3}$ → 56 trips

**30** The number of all the possible selections which a student can make for answering one or more questions out of 10 given questions in a paper, when each question has an alternative is :

(a) 1345 (b) 23560 (c) 541340 (d) 59048

**31** The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is :

(a) $^{15}C_{10}$ (b) $^{16}C_{10}$

(c) $^{16}C_{9}$ (d) none of these

**38** The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \leq n$ (where $n$ is a positive integer) is :

(a) ${}^{n+3}C_3$ (b) ${}^{n+2}C_3$ (c) ${}^{n+4}C_4$ (d) ${}^{n+4}C_3$

$x + y \geq 4$

$x + y - z = 4$

$x + y \leq 5$

$x + y + z = 5$

$x_5 \rightarrow$ is bringing the gap

$x_1 + x_2 + x_3 + x_4 + x_5 = n$

${}^{n+5-1}C_{5-1} \Rightarrow {}^{n+4}C_4$

Sq → $1^2+2^2+\dots+n^2$
$= \frac{n(n+1)}{2}$

**40** Number of rectangles on a chessboard is :
(a) 1008 (b) 1296 (c) 1124 (d) 1600

Rectangles in a Square

$\cancel{mC_2 \times nC_2} \Rightarrow$
$1^3+2^3+\dots+8^3 = \left(\frac{8\times 9}{2}\right)^2 = 1296$

$mC_2 \times nC_2$

$\Rightarrow 9C_2 \times 9C_2 \Rightarrow \frac{9\times 8 \times}{2\times 1} \Rightarrow 36^2 = 1296$

$8_2 \times {}^{2}C_2$

**42** The number of natural numbers which are smaller than $2\cdot10^8$ and which can be written by means of the digits 1 and 2 is :
(a) 678 (b) 786 (c) 766 (d) 677

**50** Priyanka has 11 different toys and Supriya has 8 different toys. Find the number of ways in which they can exchange their toys so that each keeps her initial number of toys.

(a) $^{19}C_{11}$ (b) $^{18}C_{10}$ (c) $^{20}C_{11}$ (d) $^{19}C_{11} - 1$

11 → 8

8 → 11

1 do 1 to Same

Ran ($^{19}C_{11} \times {}^{8}C_{8}$)

⇒ no of ways they can select the same no of toys from the collection of their Combined no of toys — no of ways each can select her original toys

$\Rightarrow ({}^{19}C_{11} \cdot {}^{8}C_{8} - 1)$

$\Rightarrow ({}^{19}C_{11} - 1)$

$=\left(19C_{11}-1\right)$

**57** How many different eight digit numbers can be formed using only four digits 1, 2, 3, 4 such that the digit 2 occurs twice?

(a) 20412 (b) 12042

(c) 25065 (d) none of these

**60** *In how many ways one black and one white rook can be placed on a chessboard so that they are never in an attacking position?*

(a) 1234 (b) 3136 (c) 9516 (d) 1024

**66** Find the number of numbers that can be formed using all the digits 1, 2, 3, 4, 3, 2, 1 only once so that the odd digits occupy odd places only.

(a) 9 (b) 16 (c) 18 (d) 27

**78** In how many ways all these coins can be distributed if all coins are different but all pots are identical?

(a) 14 (b) 21

(c) 27 (d) none of these

**79** In how many ways all these coins can be distributed such that no pot is empty if all coins are different but all pots are identical?

(a) 16 (b) 6 (c) 42 (d) 21

**80** In how many ways all these coins can be distributed such that no pot is empty if all coins are identical but all pots are different?

(a) 6 (b) 3 (c) 9 (d) 27