



3. Let A be a 4×4 matrix with real entries such that $-1, 1, 2, -2$ are its eigenvalues. If $B = A^4 - 5A^2 + 5I$ where I denotes the 4×4 identity matrix, then which of the following statements are correct?

1. $\det(A+B) = 0$ 2. $\det(B) = 1$ 3. trace of $A-B$ is 0 4. trace of $A^{-1}B = 4$

Handwritten work:

$$A^4 = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 2 & \\ & & & -2 \end{bmatrix} = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 2 & \\ & & & -2 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 4 & \\ & & & 5 \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 16 & \\ & & & 16 \end{bmatrix}$$

$$= A^4 - 5A^2 + 5I = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 16 & \\ & & & 16 \end{bmatrix} - 5 \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 4 & \\ & & & 4 \end{bmatrix} + 5 \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 2 & \\ & & & -2 \end{bmatrix} + \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} 0 & & & \\ & 2 & & \\ & & 3 & \\ & & & -1 \end{bmatrix}$$

$$\det(A+B) = 0$$

$$\text{tr}(A+B) = 4$$

$$A-B = \begin{bmatrix} -2 & & & & \\ & 0 & & 0 & 0 \\ & & 0 & & 0 \\ & & & 1 & 0 \\ & & & & -3 \end{bmatrix}$$

$$\text{TR}(A-B) = \underbrace{-4}_{\substack{\text{L} \quad 0 \quad 0 \quad 0 \quad \rightarrow}} \neq 0$$