90623
95723

different algorithm in statistics

Relationship is same

$\boxed{\log 10 > 10}$

$\log_{10} 100 > \log_{10} 10$

$\log_{10} 10^2 > \log_{10} 10$

$2 \log_{10} 10 > \log_{10} 10$

$\boxed{2 > 1}$

domination

B/G
H/W
B/S
S/S
B/B

RANDOMness

4, 5, 19, 23 ...

4, 100, 163, 199, 200 ...

4, −10, 7, 15, −11, ...

4, e^{x^2} $x+w$, $\ln x$, $\lg y^2$

Jan x
ISS x
Start for UPSC

Table

t, z, χ^2, F

Gym → 1000

400 — 800 — 1200 — 5000 — 10000

15,000

300

Data

1000 x3 => 3000

20 + 5 → 25

Premature Analysis

0.15

3/

150

300

150

2

all Random

Equally random

Students

6 countries

Youtube → Shorts

1 hr

1 hr

10

3/15

1

30

1 hr

20 20 20

Big data

ChatGPT

Sep 2021

BARD

ISS

$\sum x^2$

Bayes

9/K

1/37

ISS 2023

9.13

Statistics
20.7.23 Q...

Statistics
20.7.23 Q...

. Let $x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 3$ and $x_5 = 0$ be the observed values of a random sample of size 5 from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \frac{\theta}{3}, & \text{if } x = 0, \\ \frac{2\theta}{3}, & \text{if } x = 1, \\ \frac{1-\theta}{2}, & \text{if } x = 2, 3, \end{cases}$$

where $\theta \in [0, 1]$ is the unknown parameter. Then the maximum likelihood estimate of θ is
(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{5}{7}$ (d) $\frac{5}{9}$.

$$L(\theta) = \theta/3 \cdot 2\theta/3 \cdot \frac{1-\theta}{2} \cdot \frac{1-\theta}{2} \cdot \theta/3 = \frac{\theta^3(1-\theta)^2}{54} \quad \theta\in(0,1)$$

$$\log L(\theta) = 3\ln\theta + 2\ln(1-\theta) - \ln 54$$

$$l'(\theta) = \frac{3}{\theta} - \frac{2}{1-\theta} = 0 \qquad \theta = 3/5$$

$$l''(\theta) = -3/\theta^2 - \frac{2}{(1-\theta)^2} < 0 \ \forall \text{ here}$$

$\boxed{\hat\theta_0}$

$$L(\theta) = \prod_{i=1}^{3} f(x_i,\theta) = \frac{1}{\theta^3} e^{-\sum x_i/\theta}$$

$= 0$, otherwise

2) 59. Let $x_1 = 1.1$, $x_2 = 2.2$ and $x_3 = 3.3$ be the observed values of a random sample of size three from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in \Theta = \{1, 2, \ldots\}$ is the unknown parameter. Then the maximum likelihood estimate of θ equals

3. Let X_1, X_2, X_3 and X_4 be i.i.d. discrete random variables with the probability mass function

$$P(X_1 = n) = \begin{cases} \frac{3^{n-1}}{4^n}, & \text{if } n = 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(X_1 + X_2 + X_3 + X_4 = 6)$ equals ________.

$P(X_1 = n) = \frac{1}{4}\left(\frac{3}{4}\right)^{n-1}$

NBD

$NB(4, \frac{1}{4})$

$P(Y=y) = {}^{y-1}C_{4-1} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{y-4}$

Required prob..

$P(Y=6) = {}^{6-1}C_{4-1} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{6-4}$

$= {}^5C_3 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^2 = \frac{45}{2048}$

4. Let X be a random variable with the probability mass function

$$P(X=n) = \begin{cases} \frac{1}{10}, & \text{if } n = 1, 2, \ldots, 10, \\ 0, & \text{otherwise.} \end{cases}$$

Then $E(\max\{X, 5\})$ equals 6.5.

$$E[\max(X,5)] = \sum [\max(n,5)]\, P(X=n)$$

$$= \sum (\max(n,5)) \frac{1}{10}$$

$$= \frac{1}{10}\left(\sum_{n=1}^{5} \max(n,5) + \sum_{6}^{10} \max(n,5)\right)$$

$$= \frac{1}{10}\left(\sum 5 + \sum n\right.$$

$$= \frac{1}{10}(25 + 40) = 6.5$$

$$f(x|\theta) = \frac{1}{\theta^2 - \theta} \qquad \theta < x < \theta^2$$

$= 0$, otherwise

5. Let X be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \Theta = \{2, 3\}$ is the unknown parameter. For testing $H_0 : \theta = 2$ against $H_1 : \theta = 3$, let α and β be the size and power, respectively, of the test that rejects H_0 if and only if $X \geq 3.5$. Then $\alpha + \beta$ equals ________.

$$\alpha + \beta = P(\text{Rej } H_0 | H_0) + P(\text{Rej } H_0 | H_1)$$

$$= P(x \geq 3.5 \mid \theta = 2) + P(x \geq 3.5 \mid \theta = 3)$$

$$= \int_{3.5}^{\infty} f(x|2)\,dx + \int_{3.5}^{\infty} f(x|3)\,dx$$

$$= \int_{3.5}^{4} \frac{1}{2}\,dx + \int_{3.5}^{9} \frac{1}{6}\,dx$$

$$= \frac{4 - 3.5}{2} + \frac{9 - 3.5}{2} = 7/6 = \boxed{1.167}$$

Sum of the two error $= 1.167$

$P = 1 - \text{Type II Error}..$

6. Let X be an observation from a population with density

$$f(x) = \begin{cases} \lambda^2 x e^{-\lambda x}, & \text{if } x > 0, \lambda > 0, \\ 0, & \text{otherwise.} \end{cases}$$

For testing $H_0 : \lambda = 2$ against $H_1 : \lambda = 1$, the most powerful test of size α is given by "Reject H_0 if $X > c$", where c is given by

(a) $\frac{1}{4}\chi^2_{4,\alpha}$ (b) $\frac{1}{4}\chi^2_{3,\alpha}$ (c) $\frac{1}{4}\chi^2_{2,\alpha}$ (d) $\frac{1}{4}\chi^2_{1,\alpha}$.

$\alpha = P(\text{Rej } H_0 \mid H_0 \text{ is true})$

For testing $H_0 : \lambda = 2$ against $H_1 : \lambda = 1$, the most powerful test of size α is given by "Reject H_0 if $X > c$", where c is given by

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Size $\alpha = P(\text{Rej } H_0 \mid H_0 \text{ is true})$

$= P(x > c \mid \lambda = 2)$

$= \int_c^\infty 4x e^{-2x}\, dx$

Let $\boxed{4x = y}$

$= \int_{4c}^\infty y e^{-y/2} \frac{1}{4} dy$

$= \int_{4c}^\infty \frac{1}{\Gamma(4/2) 2^{4/2}} y^{4/2 - 1} e^{-y/2} dy$

$= P(\chi^2_4 > 4c)$ ∴

$4c = \chi^2_{4,\alpha}$ or, $c = \frac{1}{4}\chi^2_{4,\alpha}$

7) Ten percent of bolts produced in a factory are defective. They are randomly packed in boxes such that each box contains 3 bolts. Four of these boxes are bought by a customer. The probability, that the boxes that this customer bought have no defective bolt in them, is equal to ________.

8 Suppose that F is a cumulative distribution function, where

$$F(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1 - e^{-x}, & \text{if } 0 \leq x < 1, \\ c, & \text{if } 1 \leq x < 2, \\ 1 - e^{-x}, & \text{if } x \geq 2. \end{cases}$$

(i) Find all possible values of c.

(ii) Find $P(0.5 \leq X \leq 2.5)$ and $P(X = 1) + P(X = 2)$.

9 Let X be a random variable of continuous type with probability density function

$$f(x|\theta) = \begin{cases} \frac{\theta}{x}\left(\frac{3}{x}\right)^{\theta}, & \text{if } x > 3 \\ 0, & \text{otherwise} \end{cases} \; ; \theta > 0.$$

Based on single observation X, the most powerful test of size $\alpha = 0.1$, for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$, rejects H_0 if $X < k$. Then the value of k is

(a) 1 (b) $\frac{10}{3}$ (c) $\frac{11}{3}$ (d) 4.

10 Let $X_1, X_2, \ldots$ be a sequence of i.i.d. $U(0,1)$ random variables. If $Y_n = \sum_{i=1}^{n} X_i,\ n = 1, 2, \ldots$, then

$$\lim_{n\to\infty} P\left(Y_n \le \frac{n}{2} + \sqrt{\frac{n}{12}}\right) =$$

(a) 0.9413 (b) 0.7413 (c) 0.8413 (d) 0.6413.

31. Let S_n denote the number of heads obtained in n independent tosses of a fair coin. Using Chebyshev's inequality, the smallest value of n such that

$$P\left(\left|\frac{S_n}{n} - \frac{1}{2}\right| \leq 0.1\right) \geq \frac{3}{4},$$

is

(a) 400 (b) 200 (c) 300 (d) 100.

3. Consider the problem of testing $H_0 : \theta = 0$ against $H_1 : \theta = 1/2$ based on a single observation X from $U(\theta, \theta + 1)$ population. The power of the test "Reject H_0 if $X > \frac{2}{3}$" is
(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$.

18. Let X be a single observation from a population having an exponential distribution with mean $\frac{1}{\lambda}$. Consider the problem of testing $H_0 : \lambda = 2$ against $H_1 : \lambda = 4$. For the test with rejection region $X \geq 3$, let $\alpha = P(\text{Type I error})$ and $\beta = P(\text{Type II error})$. Then
(a) $\alpha = e^{-6}$ and $\beta = 1 - e^{-12}$ (b) $\alpha = e^{-12}$ and $\beta = 1 - e^{-6}$
(c) $\alpha = 1 - e^{-12}$ and $\beta = e^{-6}$ (d) $\alpha = e^{-6}$ and $\beta = e^{-12}$.

45. A system comprising of n identical components works if at least one of the components works. Each of the components works with probability 0.8, independent of all other components. The minimum value of n for which the system works with probability at least 0.97 is ________.

53. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with $U(0,1)$ distribution. Then

$$\lim_{n\to\infty} P\left(\sum_{i=1}^{n} X_i \leq \frac{n}{2} + n^{3/4}\right) = \_\_\_\_\_\_\_\_\_\_.$$