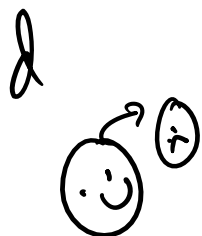


90623  
95723

difference of growth in  
Statistics

Relationship is  
same



$100 > 10$

$h_{10} 100 > h_{10} 10$

$h_{10} 10^2 > h_{10} 10$

$2 h_{10} 10 > h_{10} 10$

$271$

domination

B/G
H/W
B/S
S/S
B/B

RANDOMness

4, 5, 19, 23

4, 100, 163, 199, 200...

4, -10, 7, 15, -11, ...

$e_1, e^{x^2}, x+w, \ln x, \lg y^2$

(, )  
(, )

Table

Jan x  
ISS x  
Stat of the vpsc

t, z, n, f

Gym

5,000

1200 - 500  
= 700

300

Data

1000

X/3 => 300

Frequency Analysis

0.15

20 + 5 -> 25

3/150

300

150 2

all Random  
Equally random

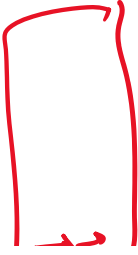
6 months

Youtube

Shorts

Subs

1 hr



~~1000~~  
~~1 hr~~  
~~(20) (20) (20)~~

1 hr  
3/5  
10  
① ③

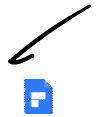
33 dam  
check apt  
6/2021

BAR  
ISS  
#  
 $\sum x^2$   
 $\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$   
ISS 2023  
Part 1/1 9.13  
20

huyefc

$\begin{matrix} \checkmark & x \\ x & \checkmark \end{matrix}$

~~Handwritten mark~~



Statistics  
20.7.23 Q...



Statistics  
20.7.23 Q...

Let  $x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3$  and  $x_5 = 0$  be the observed values of a random sample of size 5 from a discrete distribution with the probability mass function

$$f(x; \theta) = P(X = x) = \begin{cases} \theta & \text{if } x = 0, \\ \frac{2\theta}{3} & \text{if } x = 1, \\ \frac{1-\theta}{2} & \text{if } x = 2, 3, \end{cases}$$

where  $\theta \in (0, 1)$  is the unknown parameter. Then the maximum likelihood estimate of  $\theta$  is  
 (a)  $\frac{2}{3}$  (b)  $\frac{3}{5}$  (c)  $\frac{2}{5}$  (d)  $\frac{5}{9}$ .

$$L(\theta) = \theta/3 \cdot 2\theta/3 \cdot \frac{1-\theta}{2} \cdot \frac{1-\theta}{2} \cdot \theta/3 = \frac{\theta^3(1-\theta)^2}{54}$$

$$\ln L(\theta) = 3 \ln \theta + 2 \ln(1-\theta) - \ln 54$$

$$l'(\theta) = \frac{3}{\theta} - \frac{2}{1-\theta} = 0 \quad \theta = \frac{3}{5}$$

$$l''(\theta) = -\frac{3}{\theta^2} - \frac{2}{(1-\theta)^2} < 0 \quad \text{hence}$$

$\theta = \frac{3}{5}$

$$L(\theta) = \prod_{i=1}^3$$

$$f(x_i, \theta) = \frac{1}{\theta^3} e^{-\sum x_i / \theta}$$

~~= 0, otherwise~~

2) Let  $x_1 = 1.1$ ,  $x_2 = 2.2$  and  $x_3 = 3.3$  be the observed values of a random sample of size three from a distribution with the probability density function

$$f(x; \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where  $\theta \in \Theta = \{1, 2, \dots\}$  is the unknown parameter. Then the maximum likelihood estimate of  $\theta$  equals

3. Let  $X_1, X_2, X_3$  and  $X_4$  be i.i.d. discrete random variables with the probability mass function

$$P(X_1 = n) = \begin{cases} \frac{3^{n-1}}{4^n}, & \text{if } n = 1, 2, \dots, \\ 0, & \text{otherwise.} \end{cases}$$

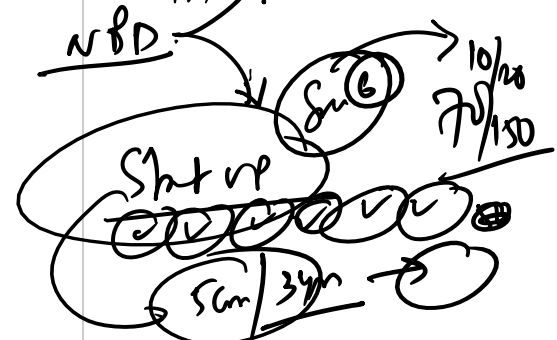
Then  $P(X_1 + X_2 + X_3 + X_4 = 6)$  equals \_\_\_\_\_

$\frac{1}{4}$   $P(X_1 = n) = \frac{1}{4} \left(\frac{3}{4}\right)^{n-1}$   $\frac{1}{4} \rightarrow \frac{3}{4}$

NBD

NBD  $(n, \frac{1}{4})$

$$P(Y=9) = \binom{4-1}{4-1} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{4-4}$$



Required prob..

$$P(Y=6) =$$

$$\binom{6-1}{4-1} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{6-4}$$

$$= \binom{5}{2} \left(\frac{1}{4}\right)^4 \left(\frac{3}{2}\right)^2 = \frac{45}{2048}$$

$$E(\max(X, 5)) = \sum \{\max(n, 5)\} P(X=n)$$

$$= \sum (\max(n, 5)) \frac{1}{10}$$

Let  $X$  be a random variable with the probability mass function

$$P(X=n) = \begin{cases} \frac{1}{10}, & \text{if } n = 1, 2, \dots, 10, \\ 0, & \text{otherwise} \end{cases}$$

Then  $E(\max(X, 5))$  equals

$$\frac{6.5}{10} \left( \sum_{n=1}^5 \max(n, 5) + \sum_{n=6}^{10} \max(n, 5) \right)$$

$$= \frac{1}{10} \left( \sum 5 + \sum n \right)$$

$$= \frac{1}{10} (25 + 60) = \underline{6.5}$$

$$f(x|\theta) = \frac{1}{\theta^2 - 0} \quad 0 < x < \theta^2$$

$$= 0, \text{ otherwise}$$

5

Let  $X$  be a sample observation from  $U(\theta, \theta^2)$  distribution, where  $\theta \in \Theta = \{2, 3\}$  is the unknown parameter. For testing  $H_0: \theta = 2$  against  $H_1: \theta = 3$ , let  $\alpha$  and  $\beta$  be the size and power, respectively, of the test that rejects  $H_0$  if and only if  $X \geq 3.5$ . Then  $\alpha + \beta$  equals

$$\alpha + \beta = P(\text{Rej } H_0 | H_0) + P(\text{Acj } H_0 | H_1)$$

$$= P(X \geq 3.5 | \theta = 2) + P(X \geq 3.5 | \theta = 3)$$

$$= \int_{3.5}^{\infty} f(x|2) dx + \int_{3.5}^{\infty} f(x|3) dx$$

$$= \int_{3.5}^4 \frac{1}{2} dx + \int_{3.5}^9 \frac{1}{6} dx$$

$$= \frac{4 - 3.5}{2} + \frac{9 - 3.5}{6} = 7/6$$

Sum of the two error  $\rightarrow 1.167$

$P = 1 - \text{Type II Error} \dots$

6

Let  $X$  be an observation from a population with density

$$f(x) = \begin{cases} \lambda^2 x e^{-\lambda x}, & \text{if } x > 0, \lambda > 0, \\ 0, & \text{otherwise.} \end{cases}$$

For testing  $H_0: \lambda = 2$  against  $H_1: \lambda = 1$ , the most powerful test of size  $\alpha$  is given by "Reject  $H_0$  if  $X > c$ ", where  $c$  is given by

- (a)  $\frac{1}{4} \lambda^2_{1,\alpha}$  (b)  $\frac{1}{4} \lambda^2_{3,\alpha}$  (c)  $\frac{1}{4} \lambda^2_{2,\alpha}$  (d)  $\frac{1}{4} \lambda^2_{1,\alpha}$

$\rightarrow$  1 or the 1  $H_0$  is true



For testing  $H_0: \lambda = 2$  against  $H_1: \lambda = 1$ , the most powerful test of size  $\alpha$  is given by "Reject  $H_0$  if  $X > c$ ", where  $c$  is given by

- (a)  $\frac{1}{4}\lambda_{4,\alpha}$  (b)  $\frac{1}{4}\lambda_{3,\alpha}^2$  (c)  $\frac{1}{4}\lambda_{2,\alpha}^2$  (d)  $\frac{1}{4}\lambda_{1,\alpha}^2$

See  $\alpha = P(\text{Reject } H_0 | H_0 \text{ is true})$   
 $= P(X > c | \lambda = 2)$

$$= \int_c^{\infty} 4x e^{-2x} dx$$

$$= \int_{4c}^{\infty} \frac{1}{4c} y e^{-y/2} \frac{1}{4} dy$$

Let  $4x = y$

$$= \int_{4c}^{\infty} \frac{1}{\Gamma(4/2) 2^{4/2}} y^{4/2-1} e^{-y/2} dy$$

$$= P(\chi_{4}^2 > 4c)$$

$$4c = \chi_{4,\alpha}^2 \quad \text{or, } c = \frac{1}{4} \chi_{4,\alpha}^2$$

7

Ten percent of bolts produced in a factory are defective. They are randomly packed in boxes such that each box contains 3 bolts. Four of these boxes are bought by a customer. The probability, that the boxes that this customer bought have no defective bolt in them, is equal to \_\_\_\_\_.

8

Suppose that  $F$  is a cumulative distribution function, where

$$F(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1 - e^{-x}, & \text{if } 0 \leq x < 1, \\ c, & \text{if } 1 \leq x < 2, \\ 1 - e^{-x}, & \text{if } x \geq 2. \end{cases}$$

- (i) Find all possible values of  $c$ .
- (ii) Find  $P(0.5 \leq X \leq 2.5)$  and  $P(X = 1) + P(X = 2)$ .

9

Let  $X$  be a random variable of continuous type with probability density function

$$f(x|\theta) = \begin{cases} \frac{x}{2} \left(\frac{x}{2}\right)^\theta & \text{if } x > 3 \\ 0 & \text{otherwise} \end{cases}; \theta > 0.$$

Based on single observation  $X$ , the most powerful test of size  $\alpha = 0.1$ , for testing  $H_0: \theta = 1$  against  $H_1: \theta = 2$ , rejects  $H_0$  if  $X < k$ . Then the value of  $k$  is

- (a) 1 (b)  $\frac{10}{3}$  (c)  $\frac{11}{3}$  (d) 4.

10

Let  $X_1, X_2, \dots$  be a sequence of i.i.d.  $U(0,1)$  random variables. If  $Y_n = \sum_{i=1}^n X_i$ ,  $n = 1, 2, \dots$ , then

$$\lim_{n \rightarrow \infty} P\left(Y_n \leq \frac{n}{2} + \sqrt{\frac{n}{12}}\right) =$$

- (a) 0.9413 (b) 0.7413 (c) 0.8413 (d) 0.6413.

31. Let  $S_n$  denote the number of heads obtained in  $n$  independent tosses of a fair coin. Using Chebyshev's inequality, the smallest value of  $n$  such that

$$P\left(\left|\frac{S_n}{n} - \frac{1}{2}\right| \leq 0.1\right) \geq \frac{3}{4},$$

is

- (a) 400 (b) 200 (c) 300 (d) 100.

3. Consider the problem of testing  $H_0 : \theta = 0$  against  $H_1 : \theta = 1/2$  based on a single observation  $X$  from  $U(\theta, \theta + 1)$  population. The power of the test "Reject  $H_0$  if  $X > \frac{2}{3}$ " is  
(a)  $\frac{2}{9}$  (b)  $\frac{5}{6}$  (c)  $\frac{1}{3}$  (d)  $\frac{2}{3}$ .

18. Let  $X$  be a single observation from a population having an exponential distribution with mean  $\frac{1}{\lambda}$ . Consider the problem of testing  $H_0 : \lambda = 2$  against  $H_1 : \lambda = 4$ . For the test with rejection region  $X \geq 3$ , let  $\alpha = P(\text{Type I error})$  and  $\beta = P(\text{Type II error})$ . Then
- (a)  $\alpha = e^{-6}$  and  $\beta = 1 - e^{-12}$     (b)  $\alpha = e^{-12}$  and  $\beta = 1 - e^{-6}$   
(c)  $\alpha = 1 - e^{-12}$  and  $\beta = e^{-6}$     (d)  $\alpha = e^{-6}$  and  $\beta = e^{-12}$ .

45. A system comprising of  $n$  identical components works if at least one of the components works. Each of the components works with probability 0.8, independent of all other components. The minimum value of  $n$  for which the system works with probability at least 0.97 is \_\_\_\_\_.



53. Let  $X_1, \dots, X_n$  be independent and identically distributed random variables with  $U(0, 1)$  distribution. Then

$$\lim_{n \rightarrow \infty} P\left(\sum_{i=1}^n X_i \leq \frac{n}{2} + n^{3/4}\right) = \text{_____}$$