MATHS 5.30PM 14 12 23

Thursday, December 14, 2023 5:37 PM

Some Problems & Solutions from Past year ISI/CMI + Test of Mathematics

Q $\left(3417^{291}\right)^{431}$ find the last digit..

$\frac{291}{4} \rightarrow$ Remain 3

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$

$(3417)^{291}$

$\frac{291}{4}$ $4 \,|\, 291 \,|\, 72$, 28, 11, 8, 3

$\rightarrow (3417)^3 \rightarrow 7^3$

$\rightarrow 343$

$3417^{291} \rightarrow$ last digit is 3

$(3)^{431} \rightarrow 3^3 \rightarrow 27 \rightarrow 3$

$\frac{431}{4}$

Final last digit. is 7

Q.2 $\left\{(8132)^{619^{22345}} + (68)^{999}\right\} \times (67)^{333}$

$\left(2^{619^{22345}} + 8^{999}\right) \times 7^{333}$

9062395123

$\frac{619}{4}$ $\Rightarrow$

$\frac{19}{4}$

$\rightarrow \left[2^{19^{45}} + 8^{99}\right] \times 7^{33}$

$8^3 = 512$

$\rightarrow \left[(2^3)^{45} + 8^3\right] \times 7^{33}$

$\rightarrow \left[(8)^1 + 8^3\right] \times 7^1$

$\rightarrow [8 + 2] \times 7$

$\rightarrow 0 \times 7 \Rightarrow 0$

2	7
4	9
8	3
6	1

Last digit $\rightarrow 0$

2	1
4	3
6	7
8	9

$\boxed{Q.3}$ 453! ends with how many zeros???

$\boxed{90623\text{-}95123}$

$453! = 1 \times 2 \times 3 \times 4 \times \cdots 450 \times 451 \times 452 \times 453$

$\boxed{\frac{453}{5}} + \boxed{\frac{453}{5^2}} + \boxed{\frac{453}{5^3}} + \cancel{\frac{453}{5^4}} + \cancel{\frac{453}{5^5}} + \cdots$

5   25   125   625

Note

do it with denominator, numerator

NO fraction $\rightarrow 90 + 18 + 3 + 0$

$\rightarrow 111$

$\rightarrow$ 111

$$\frac{453}{25} \quad 25\,\big|\,453\,\big|\,18 \quad \frac{25}{203}$$

Q.4

$12! = 1 \times 2 \times 3 \times 4 \times 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12$

$\frac{12}{5} + \cancel{\frac{12}{25}}$

$\Rightarrow 2$

$28!$ $\quad \frac{28}{5} + \frac{30}{25}$

$6 + 1 \rightarrow 0$

Q.6 How many zeros at the end of

731!

$$\frac{731}{5} + \frac{731}{25} + \frac{731}{125} + \frac{731}{625} + \ldots$$

$146 + 29 + 5 + 1$

$\Rightarrow 181$

Q. 7 Theory of equation Based

$f(g) \to +81g^9 - 731g^6 + 625\,1g^4 + g^3 - 83g^2 + 77 = 0$

$f(-g) \to -81g^9 - 731g^6 + 1\!\!\!\!\!\!\!\!\,251g^4 - g^3 - 83g^2 + 77$

Find the nature of the Roots.

Ans: (I) Check the power ordering

(II) $+ \; - \; + \; + \; - \; +$ (4)

+4 +ve Real Roots → (4)

(III) $f(-g)$ find

$- \; - \; + \; - \; - \; +$

$\boxed{\text{3 sign changes}}$ −ve Real Roots → (3)

(III) Imaginary Roots → $9 - 4 - 3 \Rightarrow$ (2)

$\boxed{\text{Q.8}}$ $f(a) \to +98a^9 + 88a^8 - 68a^6 + 44a^3 - 999a^2 + a + 6 = 0$

+4 changes +ve Real Roots

$f(-a) \to -99a^9 + 88a^8 - 68a^6 - 44a^3 - 999a^2 - a + 6 = 0$

3 sign changes −ve Real Roots

Total → (9) Imaginary Roots $9 - 4 - 3$ → (2)

Total → 9 " Imaginary Roots 9 − 4 − 5
→ 2

TOMATO Norms

$1\times2\times3$

Q. 11 $1! + 2! + 3! + \cdots + 99!$

→ $(1+2+6+24) + 120 + 720 + \cdots$ all ends with 0

except first 4

1
2
6
24
120
720
0
0
⋮
0

3

3

11 (A)

Q. 10 $(256)^{0.16}\,(16)^{0.18}$

$(2^8)^{0.16}\,(2^4)^{0.18}$

$= 2^{4(0.32+0.18)}$

$= (2^{0})^{4(0.32+0.18)}$

$\Rightarrow 2^{4(0.5)} \Rightarrow 2^{2} = 4$

$\Rightarrow 2$

Q.31 Number of Subsets

$A = \{1, 2, 3, 4\}$ $\Rightarrow 2^{4}$

1	12	123	1234	ϕ
2	13	124	$\boxed{1}$	$\boxed{1}$
3	14	341		
4	23	342		
$\boxed{4}$	24	$\boxed{4}$		
	34			
	$\boxed{6}$			

$X = \{1, 2, \ldots, 10\}$

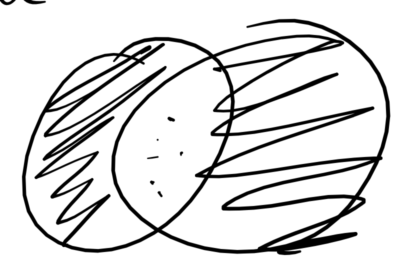
$P = \{1, 2, \ldots 3\}$ $\quad Q = \{1, 2, 4, 5\}$

$P \Delta Q = \{3\}$

$P \Delta Q = \{3\}$

There is only 1 $\rightarrow$ D

$(P - Q) \cup (Q - P)$



$(P \cup Q) - (P \cap Q)$

Symmetric difference between 2 sets ..

Equations system

$a+b+c+d+e = 63$

Find possible solutions of a, b, c, d, e

$^{63+5-1}C_{5-1}$

$\Rightarrow \boxed{^{67}C_4}$

1,1,1 (1) $a+b+c=3$

3,0,0
0,3,0
0,0,3 (3)

$^{3+3-1}C_{3-1} = {}^5C_2 = 10$

1,2,0
0,1,2
1,0,2 (6)

2

10

All solutions are non fractional

$2.5+0+0.5 \to 3$

$a+b+c+d+e+\dots+z = 63$

$a+b+c+d+e+\dots+z=63$

$63+26-1\ C_{26-1} \Rightarrow {}^{88}C_{25}$

84

$2137^{754} \rightarrow 7^{54} \rightarrow 7^{2} \rightarrow 9$

87

Remainder Calculation ..

3^{37}

79

Remainder

$3^4 \equiv 2 \pmod{79}$

$(3^4)^9 \equiv (2)^9 \pmod{79}$

$3 \cdot 3^{36} \equiv 2^9 \cdot 3 \pmod{79}$

$3^{37} \equiv 2^4 \cdot 17 \pmod{79}$

$3^{37} \equiv 2^2 \cdot (-11) \pmod{79}$

$3^{37} \equiv -44 \pmod{79}$

$3^{37} \equiv 35$

$\frac{96}{79}$

$\rightarrow 17$

$2^4=16$

$2^5=32$

$68-79$

$\rightarrow -11$

$79-44$

$\rightarrow 35$

Ans

d