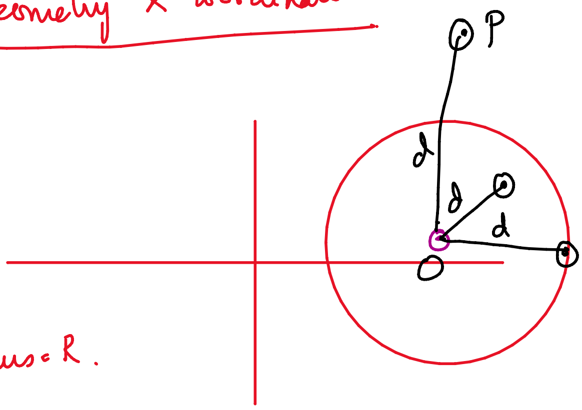
23 September 2023 15:32

Geometry & Coordinate

Let C be the circle $x^2+y^2+4x+6y+9=0$. The point $(-1,-2)$ is
(A) inside C but not the centre of C;
(B) outside C;
(C) on C :
(D) the centre of C.



$d < R$ inside
$d = R$ on
$d > R$. outside.

$(x-h)^2+(y-k)^2 = R^2$
Center (h,k) Radius R.

$(x+2)^2+(y+3)^2-13+9=0$
$(x+2)^2+(y+3)^2 = 4 = 2^2$ Center $= (-2,-3)$ Radius $= 2$.

$d = \sqrt{(-1+2)^2+(-2+3)^2} = \sqrt{2}$ $d<R$

If one vertex of an equilateral triangle of side 'a' lies at the origin and the other lies on the line $x-\sqrt{3}y=0$ then the co-ordinates of the third vertex are

(A) $(0, a)$
(B) $\left(\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$
(C) $(0, -a)$
(D) $\left(-\frac{\sqrt{3}a}{2}, \frac{a}{2}\right)$

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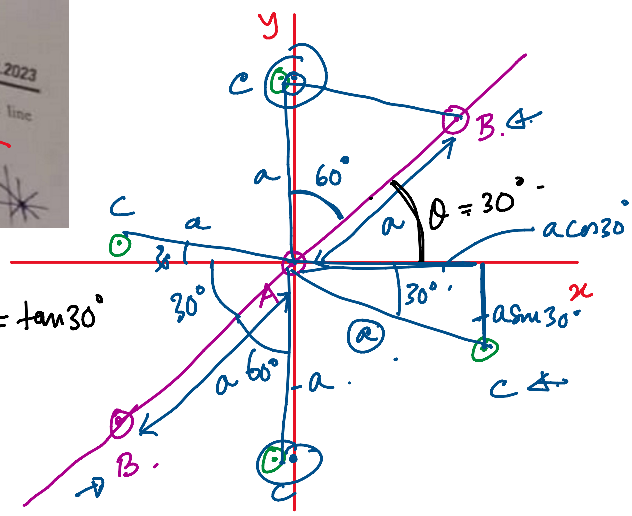
Consider the equation $y-y_1=m(x-x_1)$. If m ...

$x-\sqrt{3}y=0$.

$y = \left(\frac{1}{\sqrt{3}}\right)x$
m

$\tan\theta = m = \frac{1}{\sqrt{3}} = \tan 30°$

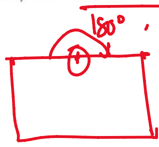
$\theta = 30°$.



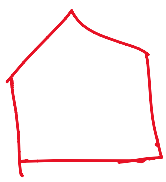
The angles of a convex pentagon are in A.P. Then, the minimum possible value of the smallest angle is
(A) ~~36°~~
(B) 36°
(C) 45°;
(D) 54°.

→ smallest .

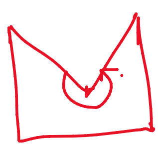
a, $a+d$, $a+2d$, $a+3d$,



Internal angle of a regular polygon



convex
all angles ≤ 180°



Concave.

(D) 54°. → smallest

a, a+d, a+2d, a+3d, a+4d. largest.

Internal angle of a polygon $= 180° - \frac{360°}{N}$.

Convex: all angles ≤ 180°. Concave.

Sum of the internal angles $= N\left(180 - \frac{360}{N}\right) = 180N - 360 = 180(N-2)$

Sum of the angles of a pentagon $= 180 \times 3 = 540°$.

$5a + 10d = 540°$

$a + 2d = 108°$ — ①.

$a + 4d < 180°$ — ②

② − ①

$2d < 72°$.

$d < 36°$.

$a > 36°$.

$d = 36°$. $a = 36°$

$a \geq 36°$

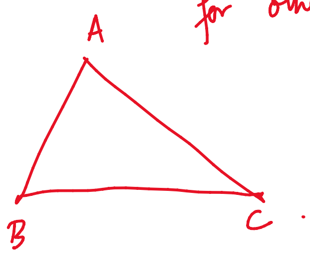
(C) (2, 1), (1, -1), (1, 2), (2, 2) (D) (-2, 1), (-1, -1), ...

Three vertices of a triangle are A(4, 3); B(1, −1) and C(7, k). Value(s) of k for which centroid, orthocentre, incentre and circumcentre of the ΔABC lie on the same straight line is/are

for equilateral Δ: all 4 points are coincident

for isosceles Δ " 4 " " collinear → Euler's line.

for other Δs (Scalene) orthocenter, centroid, circumcenter are collinear. → Euler's line



Case 1: AB = AC.

↓

$3^2 + 4^2 = 3^2 + (k-3)^2$

$(k-3)^2 = 4^2$

$k - 3 = \pm 4$

$k = 3 \pm 4$

$k = 7, -1$

Case 2: AB = BC

↓

$3^2 + 4^2 = 6^2 + (k+1)^2$

$(k+1)^2 = -11$ ✗

Case 3: AC = BC.

↓

$3^2 + (k-3)^2 = 6^2 + (k+1)^2$

$(k-3)^2 - (k+1)^2 = 27$.

$-6k + 9 - 2k - 1 = 27$

$-8k = 19$

$k = -\frac{19}{8}$

In a triangle ABC, tan A and tan B satisfy the inequality $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then which of the following must be correct?

(where symbols used have usual meanings)

(A) $a^2 + b^2 - ab < c^2$ $a^2 + b^2 - c^2 < ab$.

(B) $a^2 + b^2 > c^2$ $a^2 + b^2 - c^2 > 0$

(C) $a^2 + b^2 + ab > c^2$ $a^2 + b^2 - c^2 > -ab$.

(D) $a^2 + b^2 < c^2$ $a^2 + b^2 - c^2 < 0$.

$a^2 \; b^2 \; c^2$

↓

Cosine Rule

$\sqrt{3}x^2 - 3x - x + \sqrt{3} < 0$.

$\sqrt{3}x(x - \sqrt{3}) - 1(x - \sqrt{3}) < 0$.

$(x - \sqrt{3})(\sqrt{3}x - 1) < 0$.

$\frac{1}{\sqrt{3}} < x < \sqrt{3}$.

$\frac{1}{\sqrt{3}} < \tan A, \tan B < \sqrt{3}$

$\tan 30° < \tan A, \tan B < \tan 60°$.

$A = B = 30°$

$C = 120°$

$A = B = 60°$

$\boxed{60° < C < 120°}$

$C = 120^\circ$

$A = B = 60^\circ$

$c = 60^\circ$

$(x-\sqrt{3})(\sqrt{3}x-1) < 0$

$\sqrt{3}$

$\boxed{60^\circ < C < 120^\circ}$

$\tan 30^\circ < \tan A, \tan B < \tan 60^\circ$

$\boxed{C > A, B}$

$\boxed{30^\circ < A, B < 60^\circ}$

$\boxed{c > a, b}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$2ab\cos C = a^2 + b^2 - c^2$

$-\frac{1}{2} < \cos C < \frac{1}{2}$

$-ab < a^2 + b^2 - c^2 < ab$

$-2ab\frac{1}{2} < 2ab\cos C < 2ab \cdot \frac{1}{2}$

$-ab < 2ab\cos C < ab$

$\angle C \to$ greatest angle

$c^2 = a^2 + b^2 - 2ab\cos C$ — acute / right angle / obtuse

1. acute $\angle C < 90^\circ \to \cos C > 0$

   $a^2 + b^2 - c^2 = 2ab\cos C > 0$

   $a^2 + b^2 > c^2$

2. right angled $\angle C = 90^\circ \to \cos C = 0$

   $a^2 + b^2 - c^2 = 0$

   $a^2 + b^2 = c^2$

3. obtuse angled $\angle C > 90^\circ \to \cos C < 0$

   $a^2 + b^2 - c^2 < 0 \to a^2 + b^2 < c^2$

21. If the image of the straight line $x - 7y + 5 = 0$ (L_2) with respect to the line $x + 3y - 2 = 0$ (L_1) is represented as $ax + by + c = 0$, then $a + b + c$ equals to. (where 'a' is prime number)

A building with ten storeys, each storey of height 3 metres, stands on one side of a wide street. From a point on the other side of the street directly opposite to the building, it is observed that the three uppermost storeys together subtend an angle equal to that subtended by the two lowest storeys. The width of the street is

(A) $6\sqrt{35}$ metres;
(B) $6\sqrt{70}$ metres;
(C) 6 metres;
(D) $6\sqrt{3}$ metres.

Two circles touch each other at P. The two common tangents to the circles, none of which pass through P meet at E. They touch the larger circle at C and D The larger circle has radius 3 units and CE has length 4 units. Then the radius of the smaller circle is

(A) 1 ;
(B) $\frac{5}{7}$;
(C) $\frac{3}{4}$;
(D) $\frac{1}{2}$.

A triangle ABC has a fixed base BC. If $AB : AC = 1 : 2$, then the locus of the vertex A is

(A) a circle whose centre is the midpoint of BC;
(B) a circle whose centre is on the line BC but not the midpoint of BC;
(C) a straight line;
(D) none of the above.

Let ABC be a right angled triangle with $AB > BC > CA$. Construct three equilateral triangles BCP, CQA and ARB, so that A and P are on opposite sides of BC : B and Q are on opposite sides of CA; C and R are on opposite sides of AB. Then

(A) $CR > AP > BQ$

(B) $CR < AP < BQ$

(C) $CR = AP = BQ$

(D) $CR^2 = AP^2 + BQ^2$.