r.s : $X_1, X_2, \ldots, X_n \overset{iid}{\sim} f(\mu, \sigma^2)$ μ, σ^2 are unknown

sample mean $\bar{x} = \frac{1}{n}\sum X_i$ ...... estimator of μ.

sample var $s'^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$ ... estimator of σ^2.

With diff samples from the same popln $\Rightarrow$ diff values of $\bar{x}$ will be obtained. This gives us the sampling distribution of $\bar{x}$.

We know : $E(\bar{x}) = \mu$.

Compute
$$Var(\bar{x}) = Var\left(\frac{1}{n}\sum X_i\right)$$
$$= \frac{1}{n^2} Var\left(\sum X_i\right)$$
$$= \frac{1}{n^2}\Big[\sum Var(X_i) + \sum_{i \neq j}\sum cov\underbrace{(x_i, x_j)}_{=0}\Big].$$
$$= \frac{1}{n^2}\Big[\sum Var(x_i)\Big] = \frac{1}{n^2}\sum\sigma^2 = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}.$$

$\therefore$ Sampling distn of $\bar{x}$: $\bar{x} \sim g\left[\mu, \frac{\sigma^2}{n}\right]$.

Note: If $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

constructed r.v

[Any linear combination of normal variates will follow a normal distn with corresponding mean & var]

Eg: 2 independent normal var: $X_1 \sim N(\mu_1, \sigma_1^2)$ $X_2 \sim N(\mu_2, \sigma_2^2)$

Define: $X_3 = aX_1 + bX_2$ ... [linear combination of X_1, X_2]

$X_3 \sim N\left[E(X_3), Var(X_3)\right]$.

$\bar{X} = \frac{1}{n}\sum X_i = \frac{X_1 + X_2 + \cdots + X_n}{n} = \left(\frac{1}{n}\right)X_1 + \left(\frac{1}{n}\right)X_2 \ldots$

$$\bar{x} = \frac{1}{n}\sum x_i = \frac{x_1 + x_2 + \cdots + x_n}{n} = \underbrace{\left(\tfrac{1}{n}\right)x_1 + \left(\tfrac{1}{n}\right)x_2 + \cdots + \left(\tfrac{1}{n}\right)x_n}_{\text{Linear combination.}}$$

Normal distn and its Derivatives:

r.s: $x_1, x_2, \ldots, x_n \overset{iid}{\sim} N(\mu, \sigma^2)$.

$\Rightarrow x_i \sim N(\mu, \sigma^2) \ \forall i$.

(i) Standard Normal distribution:

$$\tau_i = \left(\frac{x_i - \mu}{\sigma}\right) \sim N(0,1) \qquad \tau_i \in (-\infty, \infty)$$

constructed r.v

(ii) Chi-square Distribution: $\chi^2 \in (0, \infty)$

$$Y = \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2 \sim \chi^2_{(n)}$$

[Chi-sq distn with 'n' degrees of freedom].

constructed r.v

$\Rightarrow$ Positively skewed distn.

d.f = total no. of independent variables

= total no. of variables − total no. of restrictions.

$$Y = \left(\frac{x_1-\mu}{\sigma}\right)^2 + \left(\frac{x_2-\mu}{\sigma}\right)^2 + \cdots + \left(\frac{x_n-\mu}{\sigma}\right)^2$$

$\Rightarrow$ n-independent terms.

$\Rightarrow$ d.f $= (n)$

Define $Y' = \sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\sigma}\right)^2 \sim \chi^2_{(n-1)}$

$$Y' = \underbrace{\left(\frac{x_1-\bar{x}}{\sigma}\right)^2} + \underbrace{\left(\frac{x_2-\bar{x}}{\sigma}\right)^2} + \cdots + \underbrace{\left(\frac{x_n-\bar{x}}{\sigma}\right)^2}$$

$\Rightarrow$ no. of terms $= n$

$\Rightarrow$ no. of restrictions $= 1$

$\sum(x_i - \bar{x}) = 0$

No. of restrictions = 1

$\boxed{\Sigma(x_i - \bar{x}) = 0}$

Num: $(x_1 - \bar{x}), (x_2 - \bar{x}), \ldots, (x_n - \bar{x}) \Rightarrow df = (n-1)$

$$\Sigma(x_i - \bar{x}) = 0$$

$$\Rightarrow (x_n - \bar{x}) = -(x_1 - \bar{x}) - (x_2 - \bar{x}) - \ldots$$

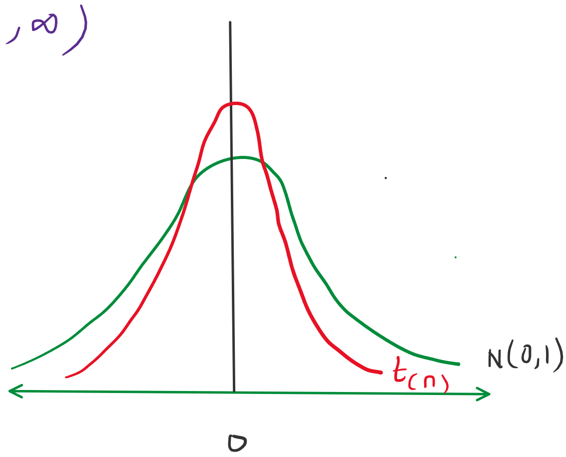
(iii) t- distribution:

$t \in (-\infty, \infty)$

$$t = \frac{z_i}{\sqrt{\chi^2_{(n)}/n}} \sim t_{(n)}$$

↳ constructed r.v.

Symmetric about zero.



(iv) F-distribution:

$F \in (0, \infty)$

Consider 2-chi-sq variates $\chi^2_{(n_1)}$ and $\chi^2_{(n_2)}$ with n_1 & n_2 d.f respectively.

$$F = \frac{\chi^2_{(n_1)}/n_1}{\chi^2_{(n_2)}/n_2} \sim F_{n_1, n_2}. \quad [\text{F-distn with } (n_1, n_2) \text{ d.f}].$$

↳ constructed r.v.

F-distn is positively skewed.