Q. If $f'(a) = f''(a) = f'''(a) = 0$ but $f^{(4)}(a) > 0$ and $f^{(4)}(x)$ is continuous at $x = a$, Then $x = a$ is a pt of:

(a) Local Maxima (c) Inflexion

~~(b)~~ Local Minima (d) None.

From defn Max/Min, even order derivative +ve ⇒ pt of Minima.

Use Taylor series to expand $f(x)$ around pt $x=a$:

$$f(x) = f(a) + \frac{(x-a)}{1!}\underbrace{f'(a)}_{=0} + \frac{(x-a)^2}{2!}\underbrace{f''(a)}_{=0} + \underbrace{\cdots\cdots}_{=0} + R_n.$$

$$f(x) = f(a) + \frac{(x-a)^4}{4!} f^{(4)}(a)$$

$$f(x) - f(a) = \frac{(x-a)^4}{4!} \underbrace{f^{(4)}(a)}_{>0} > 0 \Rightarrow f(x) - f(a) > 0 \ \forall x$$

↳ a is a pt of local minima.

Q. Let $f(0) = 0$. Show that $\lim_{h \to 0} \left\{ \frac{f(h) + f(-h)}{h^2} \right\} = f''(0)$.

Using the Taylor series expansion for $f(x)$ till second order derivative:

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a)$$

Put $a = 0 \Rightarrow f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0).$

$=0$

$$\ldots \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0).$$

For $f(h)$: Put $x = h \Rightarrow f(h) = \underbrace{f(0)}_{=0} + \frac{h}{1!} f'(0) + \frac{h^2}{2!} f''(0)$

For $f(-h)$: Put $x = -h \Rightarrow f(-h) = \underbrace{f(0)}_{0} - \frac{h}{1!} f'(0) + \frac{h^2}{2!} f''(0).$

$$\therefore \begin{cases} f(h) = h \cdot f'(0) + \frac{h^2}{2} f''(0). \\ f(-h) = -h \cdot f'(0) + \frac{h^2}{2} \cdot f''(0) \end{cases}$$

$$\frac{f(h) + f(-h)}{h^2} = f''(0).$$ [Now apply $\lim_{h \to 0}$].

Q. The coeff of x^n in the expansion of:

$\frac{(x+1)}{1!} + \frac{(x+1)^2}{2!} + \ldots\ldots$ is :

(a) e

(b) $\frac{e}{n}$

(c) $\frac{1}{n!}$

~~(d)~~ $\frac{e}{n!}$

$$\begin{cases} e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\ldots \\ \log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots \end{cases}$$

$$e^{x+1} = 1 + \underbrace{\frac{(x+1)}{1!} + \frac{(x+1)^2}{2!} + \ldots\ldots}$$

$$\frac{(x+1)}{1!} + \frac{(x+1)^2}{2!} + \ldots\ldots = (e^{x+1} - 1)$$

$$e^{x+1} - 1 = e \cdot e^x - 1.$$

$$= e\left[1 + \overset{(e)}{\frac{x}{1!}} + \overset{(e)}{\frac{x^2}{2!}} + \ldots + \overset{(e)}{\frac{x^n}{\ }} + \ldots\right] - 1$$

$$= e\left[1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots\right] - 1$$

$$= (e-1) + \left(\frac{e}{1!}\right)x + \cdots + \left(\frac{e}{n!}\right)x^n + \cdots$$

Q. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2} \cdot \log_e 3^k = ?$

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2} k \cdot \log_e 3 = \log_e 3 \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}$$

$$= \log_e 3 \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

$$= \log_e 3 \cdot \log_e 2$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad ; \quad -1 < x \le 1$$

$$x = 1 \Rightarrow \log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

Q. Sum of the series : $\frac{1}{1!} + \frac{4}{2!} + \frac{7}{3!} + \frac{10}{4!} + \cdots$ is :

(a) $(e+1)$ (b) $(e-1)$ (c) $(e+2)$ (d) $(e-2)$

$$= \underset{t_1}{\frac{1}{1!}} + \underset{t_2}{\frac{4}{2!}} + \underset{t_3}{\frac{7}{3!}} + \underset{t_4}{\frac{10}{4!}} + \cdots$$

$(2\times3-2)$ $(3\times3-2)$ $(4\times3-2)$

$$t_n = \frac{(3n-2)}{n!}$$

$\sum^{\infty}$ $\sum^{\infty} (3n-2)$ $\sum^{\infty}$ $\sum^{\infty}$

$$\sum_{n=1}^{\infty} t_n = \sum_{n=1}^{\infty} \frac{(3n-2)}{n!} = 3 \cdot \sum_{n=1}^{\infty} \frac{n}{n!} - 2 \sum_{n=1}^{\infty} \frac{1}{n!}$$

$$= 3 \sum_{n=1}^{\infty} \frac{1}{(n-1)!} - 2 \sum_{n=1}^{\infty} \frac{1}{n!}$$

$$= 3 \Big[\underbrace{1 + \frac{1}{1!} + \frac{1}{2!} + \cdots}_{e} \Big] - 2 \Big[\underbrace{\frac{1}{1!} + \frac{1}{2!} + \cdots}_{(e-1)} \Big]$$

$$= 3e - 2(e-1) = (e+2)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$$

Put $x = 1$: $1 + \frac{1}{1!} + \frac{1}{2!} + \cdots = e.$

HW.

Q. The sum of the series: $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \cdots$ is:

(a) $\frac{3e}{2}$ (b) $\frac{3e}{4}$ (c) $\frac{3e^2}{2}$ (d) $\frac{3e^2}{4}$.

$t_n =$