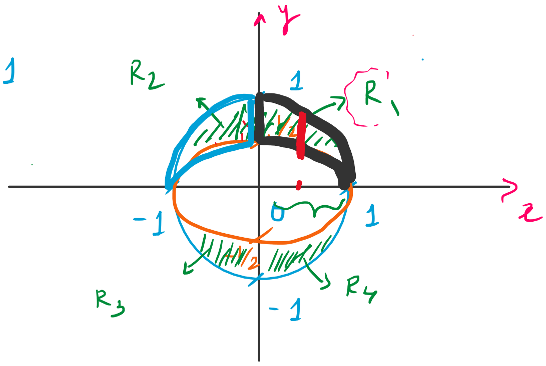
Q. Let R be the region enclosed by $x^2+4y^2 \geq 1$ and $x^2+y^2 \leq 1$. Then find $\iint_R |xy|\, dx\, dy$.

$x^2+4y^2 = 1$ --- ellipse form: $\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$ $C(\alpha,\beta)$

$x^2+y^2 = 1$ ... [Coeff of x^2 = coeff of y^2]



$C(0,0)$
$r = 1$

↳ Form of eqn of circle:
$(x-\alpha)^2+(y-\beta)^2 = r^2$
$C(\alpha,\beta)$ Radius $= r$.

$R = R_1+R_2+R_3+R_4$

As $R_1 = R_2 = R_3 = R_4$.

$R = 4R_1 = 4R_2$ ---

For ellipse: $a>b$, horizontal ellipse
$b>a$, vertical ellipse

Now: $x^2+4y^2=1 \Rightarrow \frac{x^2}{1^2}+\frac{y^2}{\left(\frac{1}{2}\right)^2}=1 \Rightarrow C(0,0)$

$$\iint_R |xy|\,dx\,dy = \iint_{R_1}\cdots + \iint_{R_2}\cdots + \iint_{R_3}\cdots + \iint_{R_4}\cdots$$

$$\iint_R |xy|\,dx\,dy = 4\iint_{R_1} |xy|\,dx\,dy.$$

$$= 4\int_0^1\int_{\frac{1}{2}\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy\ dy\ dx$$

$$= 4\int_0^1 x \int_{\frac{1}{2}\sqrt{1-x^2}}^{\sqrt{1-x^2}} y\ dy\ dx$$

$$= 4\int_0^1 x\cdot\left[\frac{y^2}{2}\right]_{\frac{1}{2}\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx$$

$$= 2\int_0^1 x\cdot\left[(1-x^2)-\tfrac{1}{4}(1-x^2)\right]dx$$

$$= 2\int_0^1 x\left[\tfrac{3}{4}-\tfrac{3}{4}x^2\right]dx$$

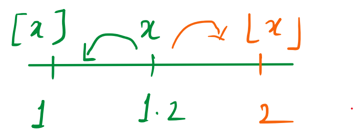
$$= 2\left(\tfrac{3}{4}\right)\int_0^1 x(1-x^2)\,dz$$

$$= \frac{3}{2}\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1 = \frac{3}{2}\left(\frac{1}{2}-\frac{1}{4}\right) = \frac{3}{2}\cdot\frac{1}{4} = \frac{3}{8}.$$

Q. Let $\lfloor x \rfloor$ denote the smallest integer $\geq x$. Find.

$$\int_0^1\int_0^1\int_0^1 \lfloor x\rfloor + \lfloor y\rfloor + \lfloor z\rfloor \, dx\, dy\, dz.$$

$\lfloor x \rfloor$ = ceiling fn.



$$\int_0^1 \lfloor x\rfloor\, dx = \int_0^1 1\, dx$$

$$\int_0^1\int_0^1\int_0^1 1 + \lfloor y\rfloor + \lfloor z\rfloor \, dx\, dy\, dz.$$

$$\int_0^1\int_0^1 \left[x + \lfloor y\rfloor\cdot x + \lfloor z\rfloor x\right]_0^1 dy\, dz$$

$$\int_0^1\int_0^1 1 + \lfloor y\rfloor + \lfloor z\rfloor \, dy\, dz.$$

$$\int_0^1\int_0^1 2 + \lfloor z\rfloor \cdot dy\, dz$$

$$\int_0^1 \left[2\cdot y + \lfloor z\rfloor\cdot y\right]_0^1 dz$$

$$\int_0^1 2 + \lfloor z\rfloor \, dz = 3.$$

Q. If the triple integral of the region bounded by the planes $2x + y + z = 4$, $x = 0$, $y = 0$, $z = 0$ is given by:

$$\int_0^2\int_0^{\lambda(x)}\int_0^{\mu(x,y)} \ldots \quad \lambda(x),\ \mu(x,y) = ?$$

$2x + y + z = 4$, $x = 0$, $y = 0$, $z = 0$ is given by :

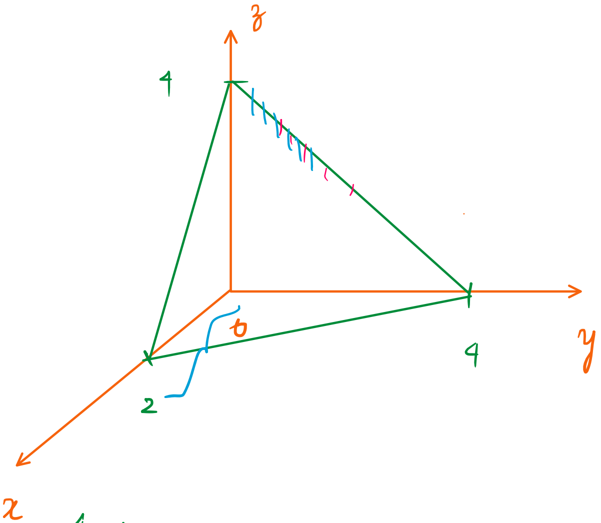
$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz\, dy\, dx, \quad \text{then} \quad \lambda(x) - \mu(x,y) = \ ?$$

(a) $x + y$ (b) $x - y$ (c) x (d) y .

$x = 0 \Rightarrow$ YZ plane

$y = 0 \Rightarrow$ XZ plane

$z = 0 \Rightarrow$ XY plane .



$2x + y + z = 4$.

$$\frac{x}{2} + \frac{y}{4} + \frac{z}{4} = 1$$

$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz\, dy\, dx = \int_0^2 \int_0^{4-2x} \int_0^{4-y-2x} dz\, dy\, dx .$$

$$\lambda(x) = 4 - 2x$$

$$\mu(x,y) = 4 - y - 2x$$

$$\lambda(x) - \mu(x,y) = \cancel{4} - \cancel{2x} - \cancel{4} + y + \cancel{2x} = y .$$

HW . Q . Let V be the region bounded by the planes $x = 0$, $x = 2$, $y = 0$, $z = 0$ and $y + z = 1$. Find $\iiint_V y \, dx\, dy\, dz$.