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Compute real and imaginary part of $z = \frac{i-4}{2i-3}$.

$$z = \frac{(i-4)}{(2i-3)} \frac{(2i+3)}{(2i+3)} = \frac{2i^2 - 5i - 12}{4i^2 - 9} = \frac{-14 - 5i}{-13}$$

$$z = \left(\frac{14}{13}\right) + \left(\frac{5}{13}\right) i$$

Write in the "trigonometric" form $(\rho(\cos\theta + i\sin\theta))$ the following complex numbers

$\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^7$.

$a + ib \to r\,\text{cis}\,\theta$.

$r = \sqrt{a^2+b^2}$ $\quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$$r = \sqrt{2}. \quad \tan\theta = \frac{\overset{-ve}{-\sin \pi/3}}{\underset{+ve}{\cos \pi/3}} = \tan\left(-\frac{\pi}{3}\right) \qquad \theta = -\frac{\pi}{3}.$$

4th Q.

$$\left[\sqrt{2}\, e^{i(-\pi/3)}\right]^7.$$

$$= (\sqrt{2})^7 e^{i\left(\frac{7\pi}{3}\right)}$$

$$= 8\sqrt{2}\left[\cos\left(\frac{7\pi}{3}\right) - i\sin\left(\frac{7\pi}{3}\right)\right].$$

$$= 8\sqrt{2}\left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right]$$

Find $z \in \mathbb{C}$ such that

$|z + 3i| = 3|z|$.

$z = a + bi$

$z + 3i = a + (b+3)i$ $\qquad |z| = \sqrt{a^2+b^2}$

$|z+3i| = \sqrt{a^2 + (b+3)^2}$ $\qquad \sqrt{a^2+(b+3)^2} = 3\sqrt{a^2+b^2}$

$a^2 + (b+3)^2 = 9a^2 + 9b^2$

$|z+3i| = \sqrt{a^2+(b+3)^2}$ $\qquad$ $\sqrt{a^2+(b+3)^2} = 3\sqrt{a^2+b^2}$

$a^2+(b+3)^2 = 9a^2+9b^2$

$8a^2 = b^2+6b+9-9b^2$

$8a^2 = 9+6b-8b^2$

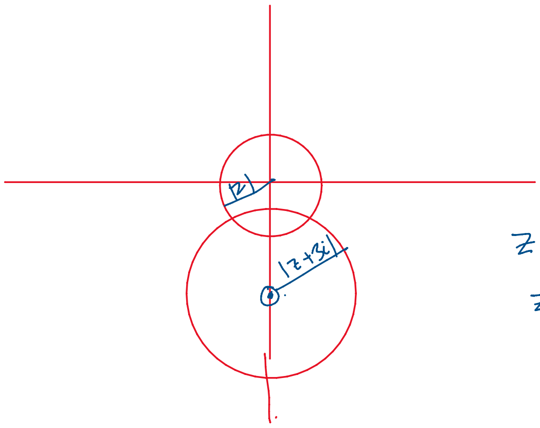
$= 9+12b-6b-8b^2$

$= 3(3+4b)-2b(3+4b)$

$= (3+4b)(3-2b)$

$$a = \pm\frac{\sqrt{(3+4b)(3-2b)}}{2\sqrt{2}}$$

$|z+3i| \Rightarrow$ circle with center $(0,-3)$ and radius $|z+3i|$



$z = x+iy$

$z+3i = x+(y+3)i$

$|z+3i| = \sqrt{x^2+(y+3)^2} = r$

$z = re^{i\theta}$

$z+3i = re^{i\theta}+3i = r(\cos\theta + i\sin\theta)+3i$

$= r\cos\theta + (r\sin\theta+3)i$

$|z+3i| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + 6r\sin\theta + 9} = \sqrt{r^2+6r\sin\theta+9}$

$3z = 3r(\cos\theta + i\sin\theta)$

$3|z| = 3r$

$r^2+6r\sin\theta+9 = 9r^2$

$8r^2 - 6r\sin\theta - 9 = 0$

$$r = \frac{6\sin\theta \pm \sqrt{36\sin^2\theta + 288}}{16}$$

$$r = \frac{6\sin\theta \pm 6\sqrt{\sin^2\theta+8}}{16} = \frac{3\sin\theta \pm 3\sqrt{\sin^2\theta+8}}{8}$$

$$z = re^{i\theta} = \left(\frac{3\sin\theta}{8} \pm \frac{3\sqrt{\sin^2\theta+8}}{8}\right)e^{i\theta} \qquad 0 \le \theta \le 2\pi$$

$z = (2+i)^{(1-i)}$ $\qquad$ $z = re^{i\theta}$

form $z = z_1^{z_2} \rightarrow (a+bi)$

$z_1 \downarrow re^{i\theta}$

$z = \left[\sqrt{5}\, e^{i\tan^{-1}(\frac{1}{2})}\right]^{(1-i)}$

$= (\sqrt{5})^{1-i}\, e^{i\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{2})}$

$= (\sqrt{5})^{1-i}\, e^{0.46+0.46i}$

$= (\sqrt{5})^{1-i}\, e^{0.46}\, e^{0.46i}$

$= \frac{\sqrt{5}}{(\sqrt{5})^i}\, e^{0.46}\left[\cos 0.46 + i\sin 0.46\right]$

$\sqrt{5}\, e^{0.46} \quad e^{i0.46}$

$\boxed{x^a = e^{\ln x^a}}$

$(\sqrt{5})^i = e^{\ln(\sqrt{5})^i} = e^{i(\ln\sqrt{5})}$

$$(\sqrt{5})^{i}$$

$$= \frac{\sqrt{5}\,e^{0.46}\cdot e^{i\,0.46}}{e^{i\ln\sqrt{5}}}$$

$$= \sqrt{5}\,e^{0.46}\,e^{i(0.46-\ln\sqrt{5})}$$

$$= \sqrt{5}\,e^{0.46}\,e^{i(-0.35)}$$

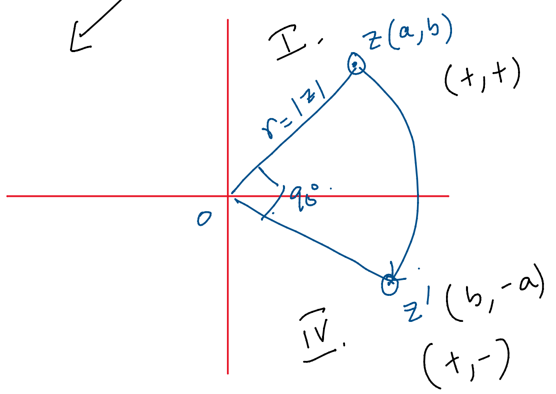
$$= \sqrt{5}\,e^{0.46}\left[\cos 0.35 - i\sin 0.35\right]$$

$$Re = \sqrt{5}\,e^{0.46}\cos 0.35$$

$$Im = -\sqrt{5}\,e^{0.46}\sin 0.35$$

Rotation of complex nos.

$z = a+bi$ → rotate by 90° clockwise around the origin



slope of $OZ = \frac{b}{a}$

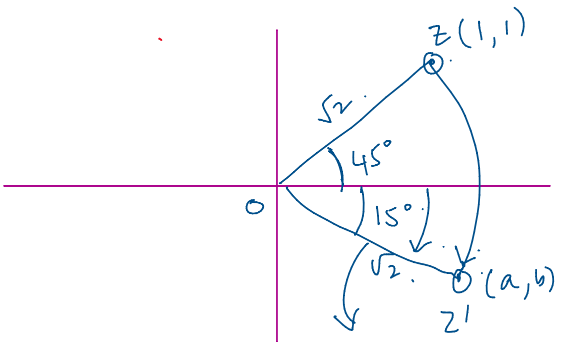
slope of $OZ' = -\frac{1}{\frac{b}{a}} = -\frac{a}{b}$

$$= -\frac{a}{b}$$

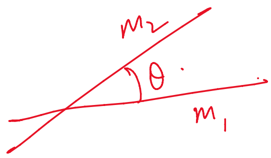
$$\underline{z' = b - ai}$$

rotate 90° anticlockwise $z' = -b + ai$

$z = 1+i$ rotate by 60° clockwise



Slope = $\tan(-15^\circ)$



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\tan 60^\circ = \frac{\tan 45^\circ - \tan(-15^\circ)}{1 + \tan 45 \tan(-15^\circ)}$$

$$\frac{b}{a} = \tan(-15^\circ) = -0.27$$

$$\boxed{b = -0.27a} \quad \text{— (1)}$$

$$\boxed{a^2 + b^2 = 2} \quad \text{— (2)}$$

Step 1 $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

Step 2 $a^2 + b^2 = r^2$

$$z^2\bar{z} + \bar{z}^2 z = 0$$

$z^2 \bar{z} + \bar{z}^2 z = 0.$