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Integration by parts

1. $15x(x+4)^{3/2}$

$= 15\left\{x\int(x+4)^{3/2}\,dx - \int\left[\frac{dx}{dx}\int(x+4)^{3/2}\,dx\right]dx\right\}$

$= x\,\frac{(x+4)^{5/2}}{5/2} - \int\left[\frac{(x+4)^{5/2}}{5/2}\right]dx$

$= \frac{2x}{5}(x+4)^{5/2} - \frac{2}{5}\int(x+4)^{5/2}\,dx$

$\frac{2x}{5}(x+4)^{5/2} - \frac{2}{5}\,\frac{(x+4)^{7/2}}{7/2}$

$= \frac{2x}{5}(x+4)^{5/2} - \frac{2}{5}\times\frac{2}{7}(x+4)^{7/2}$

$= \frac{2x}{5}(x+4)^{5/2} - \frac{4}{35}(x+4)^{7/2}$

$= \frac{2x}{5}\times\frac{3}{3}(x+4)^{5/2} - 4\times\frac{1}{35}(x+4)^{7/2}$

$= 6x(x+4)^{5/2} - \frac{12}{7}(x+4)^{7/2}$

$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$

$$\int u v\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

Q $\int 6x\,e^{x+7}\,dx$

$= 6x\int e^{x+7}\,dx - \int\left[6\int e^{x+7}\,dx\right]dx$

$= 6x\,e^{x+7} - \int 6\,e^{x+7}\,dx$

$= 6xe^{x+7} - 6e^{x+7} + c$ (ans)

$= 6e^{x+7}(x-1) + c$

Q $\int \frac{5x}{(x-1)^2}\,dx$

$= \int 5x\,(x-1)^{-2}\,dx$

$= 5x\int (x-1)^{-2}\,dx - \int\left[5\int (x-1)^{-2}\,dx\right]dx$

$= 5x\,\frac{(x-1)^{-1}}{-1} - \int 5\,\frac{(x-1)^{-1}}{-1}\,dx$

$= -\frac{5x}{(x-1)} + 5\int \frac{dx}{(x-1)}$

$= -\frac{5x}{(x-1)} + 5\ln|x-1| + c.$

$\int \frac{dx}{x} = \ln|x|$

$\int \frac{dx}{x-1} = \ln|x-1|$

$x^4(2x-5)^4$

Let $u = 2x-5$