Central Limit Theorem

Small vs Large Sample

$n < 30$ $\quad$ $n \geq 30$

depend on the Structure of the Data..

If Skewness ↑ $\quad$ $n >> 30$ is a large sample

$n = 50, 60, 70 \ldots$

very Similar / low Skew $\quad$ $n = 10 \rightarrow$ large.

Shape of the population is the decider.

CLT Thm

If $X_1, X_2 \ldots X_n \rightarrow$ iid $\quad \mu, \sigma^2$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow \text{SND if } n \rightarrow \infty$$

$$N(0,1)$$

I will have a very very very Large Sample

140 crore → The Great Indi WAVE $\quad$ 50 crore

Bose → Bose-Einstein Cond

50 $\quad$ 100 Auto

$n = 3$ $\quad$ X X X $\quad$ R_n 3

X 000 X

$n=3$ ((X X X')) n -

X 8 0 0 0 0 0 0 X 0 0 0 0 0 0 0 0 X

Run Test...

Randomness of the B me decider...

(19-23)

DPIf

19 20 21 22 23

x x x x x

$n=50$

15 15 20

65 1

62

403000

Avera... μ 2

(000 → 6200 σ^2 $\sigma = 650$

56 $n = 65$ value Total bill > 4,00,000

~~403000~~

136

415

56440

$136 \times 415 = 56440$

$Z \sim N(0,1)$

$S \sim N[65 \times 6200, 65 \cdot 650^2]$

$\sim [403000, 5240^2]$

$$P(S > 400000) = P\left(Z < \frac{400000 - 403000}{5240}\right)$$

$$= P(Z < 0.572)$$

$$= 0.71$$

Normal Approx to Bin Dis

$\sum X_i \sim B(n,p)$

$\bar{X} \sim N(\mu, \sigma^2/n)$ or $\sum X_i \sim N(n\mu, n\sigma^2)$ ..

Continuity Correction

Continuous Distributions

Poisson

change in equality $\geq$, or $\leq$ b/w consecutive integers.

2023

2020

19 Pune

20

21

22

23 Data

Themes 1500

n

$n_1 \Rightarrow 100 + 20$

8%–10%

$n_1 \rightarrow 100$ to 150

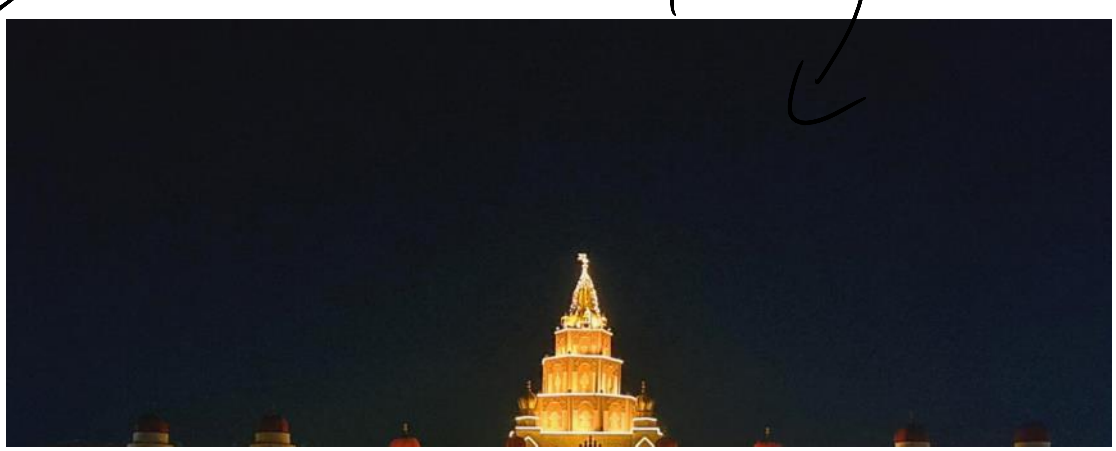
Permutations

Theme

Alphabetical

Pandal

College Square



Under Range

Q 45% people (A) BG
$n = 300$ find P → (more than 115 have blood Group A)

$X \sim Bin(300, 0.45) \doteq N(135, 74.25)$

$P(X > 115) \approx P(X > 115.5)$

$P\left(Z < \frac{115.5 - 135}{\sqrt{74.25}}\right) = P(Z > -2.263)$
$= 0.988$

Why approximate??

Salary Rising Lower

Rising Lower)

Job → Salary

Incentive

Risk Neutral

Job → High Salary

Risk Averse

CLT

$X \sim B(100, 0.6)$ find $P(10 \le x \le 16)$

$$\text{CLT} \Rightarrow \frac{X - 100 \times 0.6}{\sqrt{100 \cdot 0.6 \cdot 0.4}} = \frac{X-60}{\sqrt{24}} \to Z \to N(0,1)$$

$$P(10 \le X \le 16) = P(9.5 \le \cancel{X} \le 16.5)$$

$$= P\left(\frac{9.5-60}{\sqrt{24}} \le \frac{X-60}{\sqrt{24}} \le \frac{16.5-60}{\sqrt{24}}\right)$$

$$= P(-10.3 \le Z \le -8.88)$$

→ Table use...

Q

Limiting distribution

$$X_n \to f_n(x) = \begin{cases} \frac{1}{\Gamma n} e^{-x} x^{n-1} & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

→ of $Y_n = \frac{X_n}{n}$

Ans $X_n \sim \text{Gam}(n, 1)$

$$\frac{1}{(1-t)^n} = (1-t)^{-n}$$

$M_{x_n}(t) = \frac{1}{(1-t)^n} = (1-t)^{-n}$

Let, $Y_n = \frac{X_n}{n}$ $\quad M_{Y_n}(t) = M_{X_n}(t/n) = (1-t/n)^{-n}$

$\lim_{n\to\infty} M_{Y_n}(t) = \lim_{n\to\infty}(1-t/n)^{-n} = \lim_{n\to\infty}\frac{1}{(1-t/n)^n} = e^t$

Hence the limiting distn is e^t

\# $X_1, X_2 \ldots X_n \to U(0,2)$ If $Y_n = \bar{X}_n$

Show $\sqrt{n}(Y_n - 1) \to N(0, 1/3)$

$X_i \sim U(0,2)$ $\quad E(X_i) = 1$ $\quad V(X_i) = 1/3$

$\bar{X} \sim N\left(E(X), \frac{V(X)}{n}\right)$ $\quad \bar{X} \sim N(1, 1/3n)$

By CLT $\quad \sqrt{n}(\bar{X}_n - 1) \to N(0, 1/3)$

$\sqrt{n}(Y_n - 1) \to N(0, 1/3)$ Proved.

Q

$X_j = Z_j - Z_{j-1}$ $\quad J = 1, 2 \ldots n$

$Z_0, Z_1, \ldots Z_n$ iid with common var σ^2

$z_0, z_1, z_2 \ldots z_n$ <u>IID</u> with common var σ^2

Find var $\left(\frac{1}{n}\sum_{j=1}^{n} x_j\right) = ?$

Ans: $z_j, z_k \to$ indept

$$V(z_j - z_k) = \sigma^2 + \sigma^2 = 2\sigma^2$$

$$V\left(\frac{1}{n}\sum x_i\right) = \frac{1}{n^2} V\big((z_1 - z_0) + (z_2 - z_1) + \ldots (z_n - z_{n-1})\big)$$

$$= \frac{1}{n^2} V(z_n - z_0)$$

$$= 2\sigma^2/n^2$$

<u>Q</u> $x_1, x_2 \ldots x_{100} \to$ <u>UD</u> <u>$\sigma^2 > 0$</u>

$\phi \to$ CDF of SND

find $P\left(\sum_{i=1}^{100} x_i \le a\mu\right)$

Ans: CLT $\to$

$$P\left(\sum_{i=1}^{100} x_i \le a\mu\right) = P\left(\frac{\sum x_i}{100} \le \frac{a\mu}{100}\right)$$

$$= P\left(\bar{X} \le \frac{a\mu}{100}\right)$$

$$= P\left(\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{100}}} \le \frac{\frac{a\mu}{100} - \mu}{\frac{\sigma}{\sqrt{100}}}\right)$$

$$\Rightarrow \phi\left(\frac{\frac{a\mu}{100} - \mu}{\frac{\sigma}{\sqrt{100}}}\right)$$

$$\Rightarrow \phi\left(\frac{\frac{a\mu}{100} - \mu}{\sigma/10}\right) = \phi\left(\frac{a\mu - 100\mu}{10\sigma}\right)$$

Beta/Gamma functions

Double integral

$$= \phi\left(\frac{(a-100)\mu}{10\sigma}\right)$$

$\boxed{9062395723}$

Q $X_1, X_2 \ldots$ iid each r.v → df (4)

If $S_n = \sum_{i=1}^{n} X_i^2$ $n = 1, 2 \ldots$ If $\frac{S_n}{n} \xrightarrow{p} \mu$ as $n \to \infty$

$\mu = ?$

PYQ → ~~2016~~ 3 eq − 3 un → 0

6 eq − 2 un ⇒ 4

Solving df ≥ 0

$X_i \sim \chi^2_4$ $\forall$ $i = 1, 2, 3, 4$

$\mu = E\left(\frac{S_n}{n}\right)$

$$n = 4$$

$$S_n = \sum x_i^2 \qquad \mu = E\left(\frac{S_n}{n}\right)$$

$$E\left(\frac{S_n}{n}\right) = \frac{1}{n} E\left(\sum X_i^2\right)$$

$$= \frac{1}{n} n E(X_1^2) = V(X_1) + (E(X_1))^2$$

$$= 4^2 + 8 = 24$$

$$\boxed{\mu = 24}$$