Model: $Y_i = \alpha + \beta X_i$ (Saving) (Income) $H_0: \beta = 0$ vs $H_1: \beta \neq 0$.

Estimated model: $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$, $\hat{\alpha}, \hat{\beta}$ are estimated through OLS

Model: $Y_i = \alpha + \beta X_i + \gamma X_i^2$ ... [Quad Rel. X & Y].

Estimated model: $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i + \hat{\gamma} X_i^2$.

$$\min_{\hat{\alpha}, \hat{\beta}, \hat{\gamma}} \sum \epsilon_i^2 = \sum (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{n} (Y_i - \hat{\alpha} - \hat{\beta} X_i - \hat{\gamma} X_i^2)^2$$

$$\frac{\partial \sum \epsilon_i^2}{\partial \hat{\alpha}} = 0 \Rightarrow$$

$$\frac{\partial \sum \epsilon_i^2}{\partial \hat{\beta}} = 0 \Rightarrow$$

$$\frac{\partial \sum \epsilon_i^2}{\partial \hat{\gamma}} = 0 \Rightarrow$$

$\rightarrow$ 3 eqns to solve for 3 unknowns $\rightarrow \hat{\alpha}, \hat{\beta}, \hat{\gamma}$.

Testing of Hypothesis..

Eg: Popln: $X \sim N(\mu, 1)$ [μ = unknown popln parameter].

r.s $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, 1)$.

Objective: $H_0: \mu = 0$ vs $H_1: \mu \neq 0$.

H_0 ↓ Null Hypothesis

H_1 ↓ Alternative Hypothesis.

The testing structure will be developed based on the sample.

There 2 types of errors that can be committed:

	H_0 true	H_0 false	(True state of popln).
→ H_0 accepted	✓	Type II error	

→ H_0 accepted	✓	Type II error
→ H_0 rejected	Type I error	✓

(Result from Test)

Define α = Type I Error = $P[H_0 \text{ rejected} \mid H_0 \text{ is true}]$.

β = Type II Error = $P[H_0 \text{ accepted} \mid H_0 \text{ is false}]$.

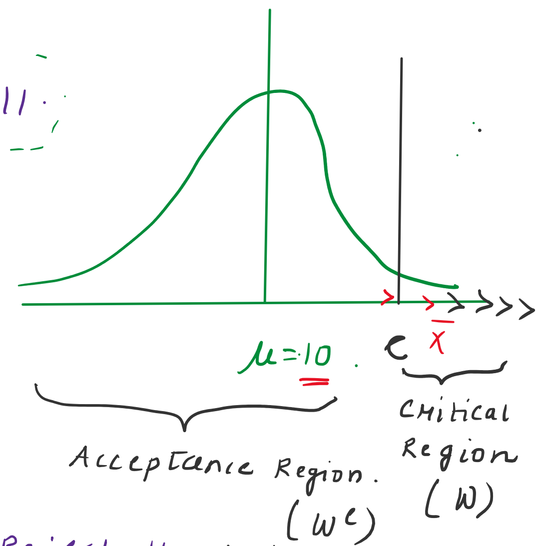
Eg: $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, 1)$.

To test: $H_0 : \mu = 10$ vs $H_1 : \mu = 11$.

$E(\bar{X}) = \mu$.

$\bar{X} \sim N\left(\mu, \frac{1}{n}\right)$.

Under H_0: $\bar{X} \sim N\left(10, \frac{1}{n}\right)$



Critical Region: If $\bar{X} > c \Rightarrow$ Reject $H_0 \Rightarrow$

If $\bar{X} \leq c \Rightarrow$ Accept H_0.

Let T be the sample statistic used to test the hypothesis then, $\alpha = P[T \in W \mid H_0]$.

$\beta = P[T \in W^c \mid H_1]$.

Q. $X \sim Exp\left(\frac{1}{\theta}\right)$. To test $H_0 : \theta = 10{,}000$

vs $H_1 : \theta = 20{,}000$.

Critical region: $X \geq 16{,}000$. Find α, β.

Bin

Poi

Uniform

Critical region: $X \geq 16,000$. Find α, β.

Uniform
Exp.
Normal.

$X \sim Exp\left(\frac{1}{\theta}\right)$

$f(x) = \frac{1}{\theta} e^{-x/\theta}$; $x \geq 0$.

$\therefore \quad \alpha = P\left[X \geq 16,000 \,/\, \theta = 10,000\right]$.

$\alpha = P[T \in W / H_0]$.

$$= \int_{16,000}^{\infty} \frac{1}{10,000} e^{-\frac{x}{10,000}} dx \quad . =$$

$\beta = P\left[X < 16,000 \,/\, \theta = 20,000\right]$.

$\beta = P[T \in W^c / H_1]$.

$$= \int_{0}^{16,000} \frac{1}{20,000} e^{-\frac{x}{20,000}} dx =$$

$$\alpha = \int_{16,000}^{\infty} \frac{1}{10,000} e^{-\frac{x}{10,000}} dx$$

$\frac{x}{10,000} = z$

$dx = 10,000 \, dz$.

$x = 16,000, \; z = 1.6$.

$x \to \infty, \; z \to \infty$.

$$= \frac{1}{\cancel{10,000}} \int_{1.6}^{\infty} e^{-z} . \, 10,\cancel{000} \, dz$$

$$= \int_{1.6}^{\infty} e^{-z} dz = -\left[e^{-z}\right]_{1.6}^{\infty}$$

$$= -\left[0 - e^{-1.6}\right] = e^{-1.6}.$$

Q. Popln: $N(\mu, 1)$. Consider a r.s of size '1' from the popln to test $H_0: \mu = 8$ vs $H_1: \mu = 12$.

Critical region: $X \geq 9$. Find α, β.

$X \sim N(\mu, 1)$

$f(x) = \frac{1}{\ldots} e^{-\frac{1}{2}(x-\mu)^2}$

$X \sim N(\mu, \sigma^2)$

$f(x) = \frac{1}{\ldots} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$X \sim N(\mu, 1)$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2} \quad -\infty < x < \infty.$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad x \in \mathbb{R}$$

$W = X \geq 9.$

$$\alpha = P[X \geq 9 \mid \mu = 8] = \int_{9}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-8)^2}\,dx$$

HW.

$$\beta = P[X < 9 \mid \mu = 12] = \int_{-\infty}^{9} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-12)^2}\,dx.$$