Limits

$$\lim_{x\to 1} \frac{\sqrt{x^2+8}-\sqrt{10-x^2}}{\sqrt{x^2+3}-\sqrt{5-x^2}}$$

Rationalisation. ans = $2/3$

$$\lim_{x\to 0} \frac{3x-\sin^{-1}(x)}{4x-\tan^{-1}(x)}$$

$$\lim_{x\to 0} \frac{3-\sin^{-1}x/x}{4-\tan^{-1}x/x}$$

$$* \quad \lim_{x\to 0} \left(1+f(x)\right)^{1/f(x)} = e$$

$$\lim_{x\to 0} \frac{b^{f(x)}-1}{f(x)} = \ln b \quad , \; b>0$$

$$\lim_{x\to a} \frac{\ln(1+f(x))}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{e^{\tan x}-e^{x}}{\tan x - x}$$

$$\lim_{x\to 0} \frac{x^n-a^n}{x-a} = na^{n-1}$$

Let m and n be two positive integers greater than 1. If $\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right) = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is

$$\lim_{\alpha\to 0} \frac{e^{\cos\alpha^n}\cdot(-\sin\alpha^n)\cdot n\alpha^{n-1}}{m\alpha^{m-1}}$$

$$= -\frac{n}{m} \lim_{\alpha\to 0} e^{\cos\alpha^n}\cdot \sin\alpha^n \cdot \alpha^{n-m}$$

$$= -\frac{n}{m}\left[\lim_{\alpha\to 0} e^{\cos\alpha^n} \cdot \lim_{\alpha\to 0} \frac{\sin\alpha^n}{\alpha^n}\cdot \alpha^{n-m}\cdot\alpha^n\right]$$

$$= -\frac{n}{m}\cdot e \cdot \boxed{\lim_{\alpha\to 0} \alpha^{2n-m}}$$

$2n-m=0$

$\Rightarrow \; m/n = 2$

$$\lim_{x\to 0} \frac{ae^x-b}{x} = 2. \quad \text{Find } a \;\&\; b$$

$a=b=2$ (use expansions)

$f(x) \le \underline{g(x)} \le h(x)$

$\lim_{x\to a} f(x) = L$

$\lim_{x\to a} h(x) = L$

$$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x) \le \lim_{x\to a} h(x)$$

$$\Rightarrow \quad L \le \lim_{x\to a} g(x) \le L$$

$$\Rightarrow \quad \lim_{x\to a} g(x) = L$$

HW $\quad \lim_{x\to\infty} \frac{\log(x)}{[x]}$

Questions

The value of $\lim_{n\to\infty} \frac{[x]+[2^2 x]+[3^2 x]+\cdots+[n^2 x]}{n^3}$ (where $[\cdot]$ represents the greatest integer function) is

(A) x (B) $\frac{x}{2}$ (C) $\frac{x}{3}$ (D) $\frac{2x}{3}$ (E) none of these

The value of $\lim_{n\to\infty} n^{-n^2}\left((n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{2^2}\right)\ldots\left(n+\frac{1}{2^{n-1}}\right)\right)^n =$

(A) 1 (B) e (C) e^2 (D) none of these

The value of $\lim_{x\to\frac{\pi}{2}} \frac{\sin x-(\sin x)^{\sin x}}{1-\sin x+\ln(\sin x)}$ is

(A) 1 (B) 2 (C) 3 (D) 4

The value of $\lim_{x\to 0^+} \left(\frac{1}{x}\right)^{\sin x}$

(A) 0 (B) 1 (C) e (D) None of these

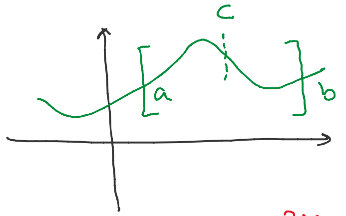
Continuity

$$\lim_{x\to\infty} \frac{\log x}{[x]}$$

$$x-1 < [x] \le x$$

$$\underbrace{\frac{\log x}{x}}_{f(x)} \le \underbrace{\frac{\log x}{[x]}}_{g(x)} < \frac{\log x}{x-1} \leftarrow h(x)$$

$$\lim_{x\to\infty} f(x) \le \lim_{x\to\infty} g(x) \le \lim_{x\to\infty} h(x)$$



Properties $\quad f(x), g(x) \to$ continuous at a

(1) $cf(x)$ is continuous at a

(2) $f(x) \pm g(x) \xrightarrow{c}$ at a

(3) $f(x)\,g(x) \xrightarrow{c}$ at a

(4) $f(x)/g(x) \xrightarrow{c}$ at a if $g(a) \neq 0$

$$\lim_{n\to\infty} \frac{\log(x+2) - x^{2n}\sin(x)}{x^{2n}+1}$$

3 phases $\qquad |x|=1,\ |x|<1,\ |x|>1$

$|x|<1$
$$\lim_{n\to\infty} \frac{\log(x+2) - x^{2n}\sin(x)}{x^{2n}+1} = \log(x+2)$$

$|x|=1$
$$\frac{\log(x+2) - \sin(x)}{2}$$

$|x|>1$
$$\lim_{n\to\infty} \frac{\frac{\log(x+2)}{x^{2n}} - \sin(x)}{1 + 1/x^{2n}} = (-\sin x)$$

$$f(x) = \begin{cases} -\sin x & x < -1 \\ \log(x+2) & -1 < x < 1 \\ -\sin x & x > -1 \\ \dfrac{\log(x+2) - \sin(x)}{2}, & x = \pm 1 \end{cases}$$

$\therefore$ at $x = \pm 1$

$f(x)$ is discontinuous

$$f(x) = [x] + [-x]$$

$$\lim_{x\to 1^+} f(x) = -1 \qquad f(1) = 0$$

$$\lim_{x\to 1^-} f(x) = -1$$

(removable)

$x \in (0,1) \cup (1,2)$

Continuous in $(0,2)$